Classifier Evaluation and Selection

Review and Overview of Methods

Considerations for Evaluating Classification Models

- Interpretation vs. Prediction

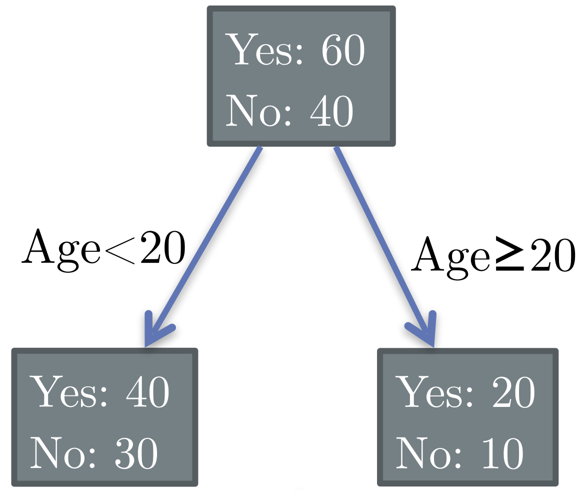
  (Model Parsimony vs. Model Error)

- Type of prediction goal:
  - **Decisions** – Interested only in **resulting classification (ex: 'Yes'/'No')** *pick out all the winning proposals*
  - **Rankings** – Interested in **ranking individuals by their 'true likelihood'** of an outcome - *who are the best 10% to market to*
  - **Estimates** – Interested in **predicting probabilities** or a continuous outcome accurately - *compute expected annual cost of each machine using failure probabilities*

Model Fit Statistics Summary

Prediction Type	Model Fit Statistics
Decisions	Lift/Gain/Profit/Loss Accuracy/ Misclassification KS-Statistic
Rankings	ROC Index concordance statistic Gini Coefficient
Estimates	Average Squared Error SBC/Likelihood MAPE R^2

Practical Difference



I(t) is **<u>misclassification rate</u>**

Parent misclassification rate: 40%

Misclassification rate after split: 40%

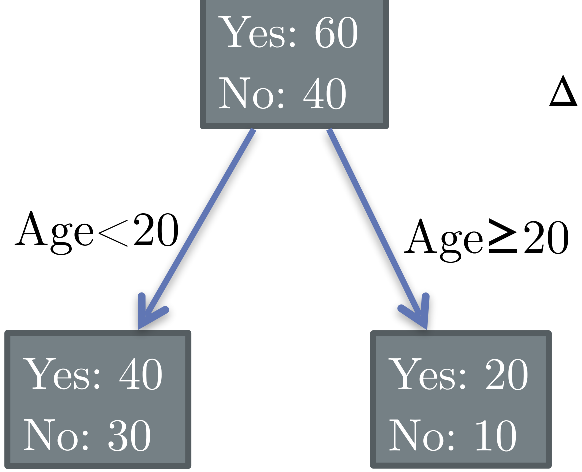
Gain: **0** => Don't make this split.

I(t) is **<u>Average Squared Error</u>**

Parent averaged squared error: 0.24

Average squared error after split: 0.23

Gain: **0.01** => Consider this split.



$$\Delta = I(t) - \left(\frac{n_L}{n} I(t_L) + \frac{n_R}{n} I(t_R)\right)$$

I(t) is **misclassification rate**

Parent misclassification rate: 40%

Misclassification rate after split: 40%

Gain: **0** => Don't make this split.

Details:

Left child misclass. rate: 42%

Right child misclass. rate: 33.3%

$$\text{Gain} = 0.40 - \left(\frac{70}{100}0.42 + \frac{30}{100}0.33\right) = 0$$

I(t) is **Average Squared Error**

Parent averaged squared error: 0.24

Average squared error after split: 0.23

Gain: **0.01** => Consider this split.

Details:

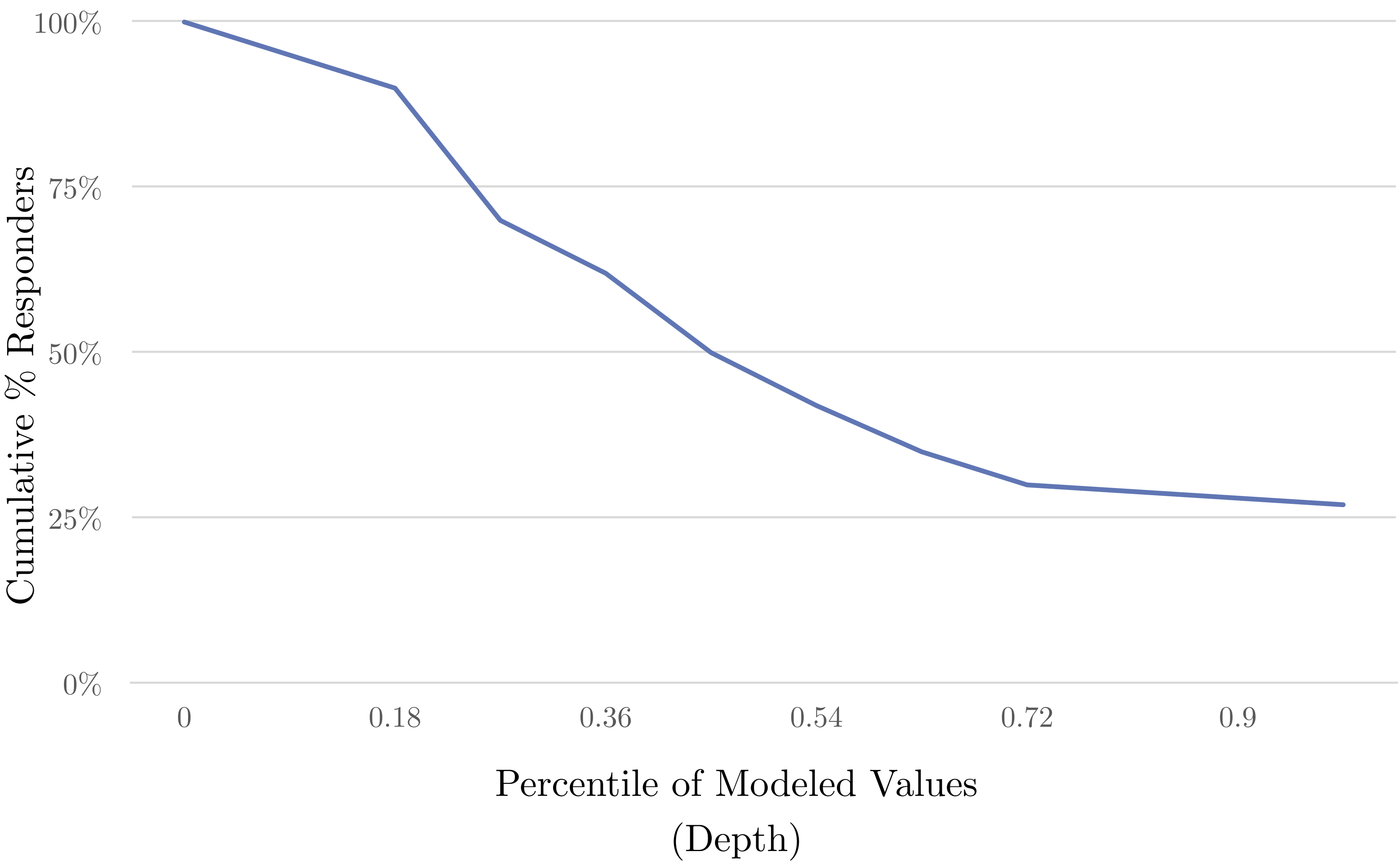
Parent ASE: $\frac{1}{100}\left(60\,(1-0.6)^2 + 40\,(0-0.6)^2\right)$

Left child ASE: $\frac{1}{70}\left(40\left(1-\frac{4}{7}\right)^2 + 30\left(0-\frac{4}{7}\right)^2\right)$

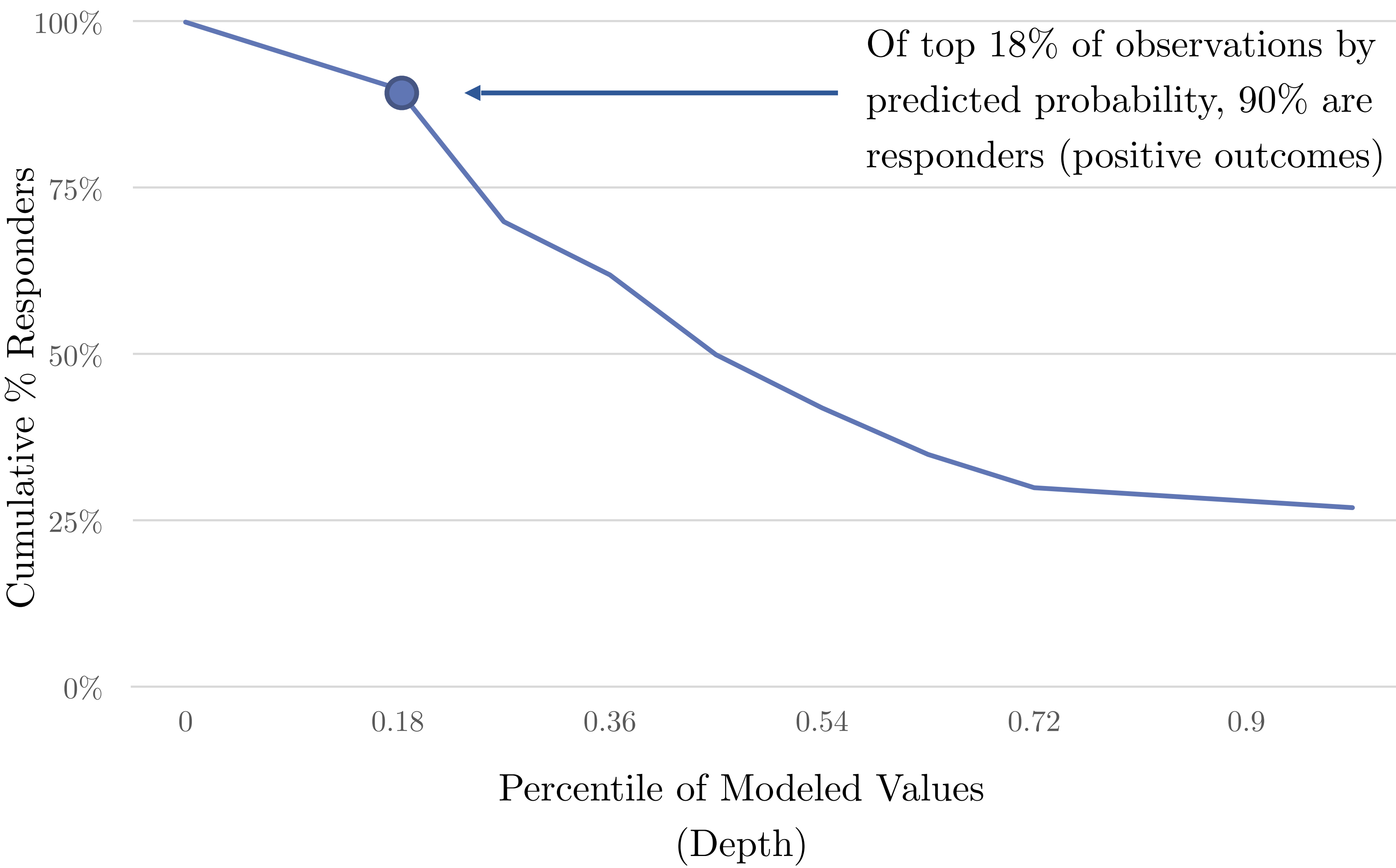
Right child ASE: $\frac{1}{30}\left(20\left(1-\frac{2}{3}\right)^2 + 10\left(0-\frac{2}{3}\right)^2\right)$

$$\text{Gain} = 0.24 - \left(\frac{70}{100}0.245 + \frac{30}{100}0.222\right) = 0.01$$

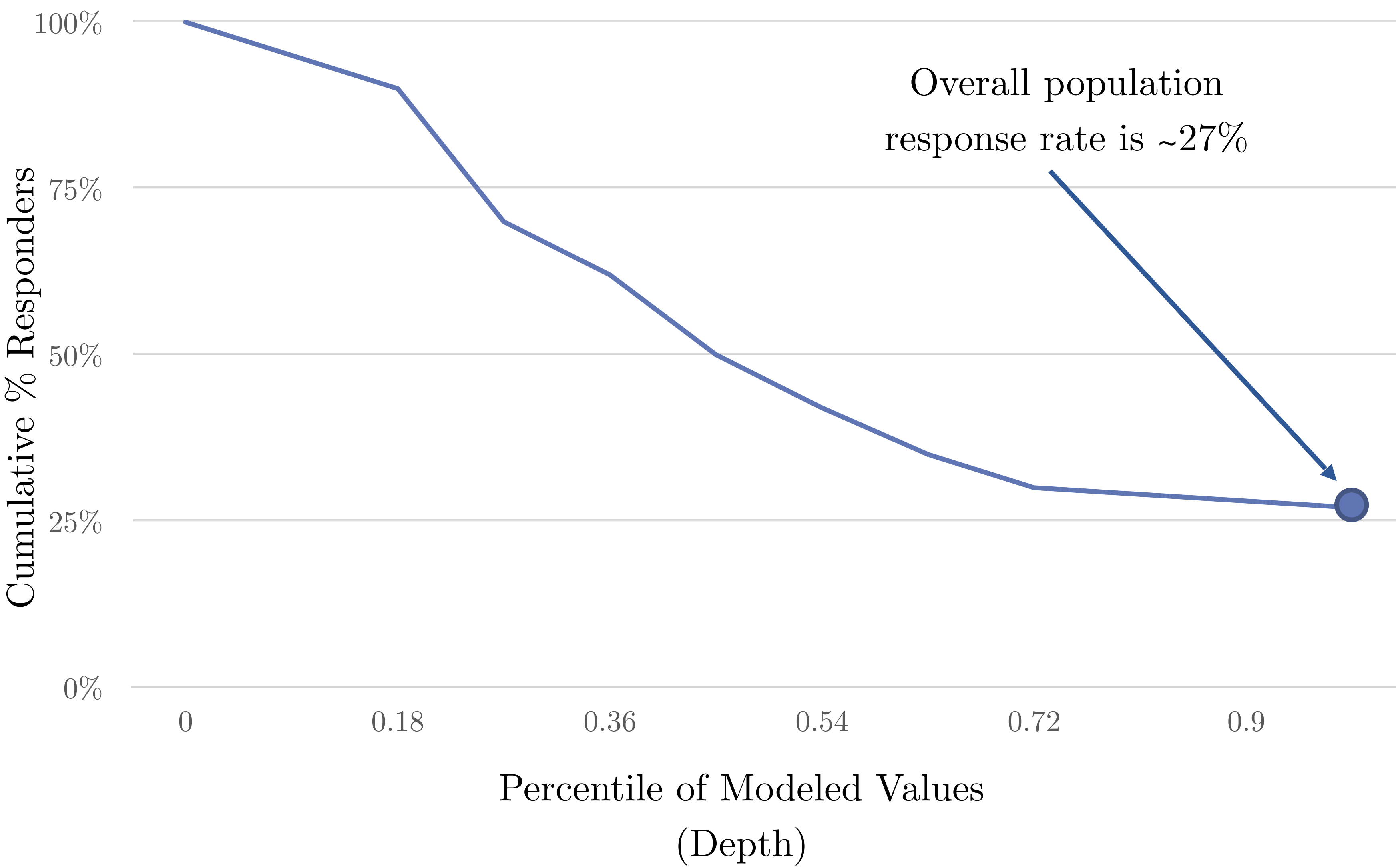
Response/Gain Charts



Response/Gain Charts



Response/Gain Charts

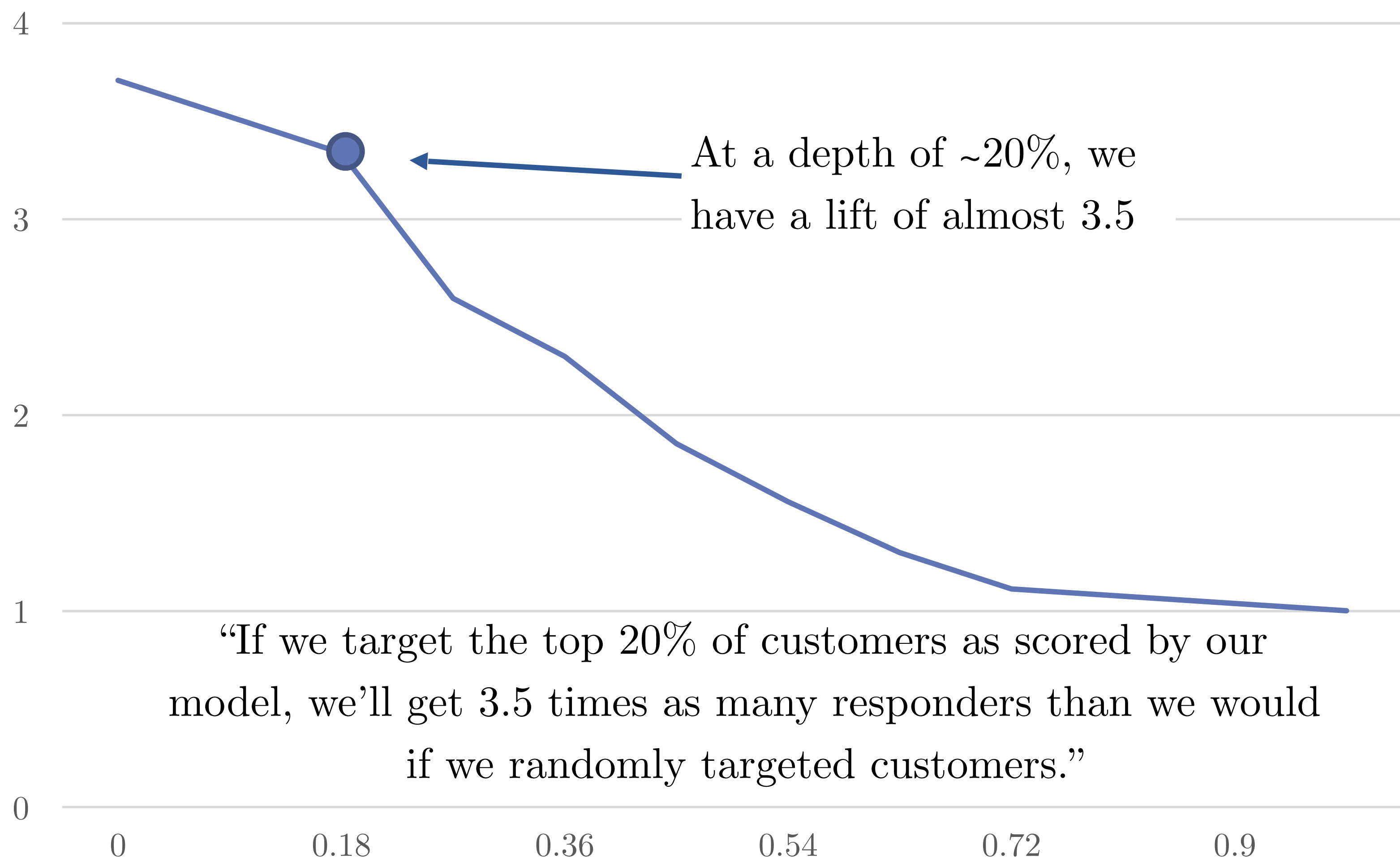


Lift Chart

While it's great to know how many responders you got in the top p% of observations scored by the model, it's *even better* to **know how your model compares to random selection.**

$$\text{Lift} = \frac{\%\ Responders\ from\ Model}{\%\ Responders\ from\ Random\ Selection}$$

Cumulative Lift



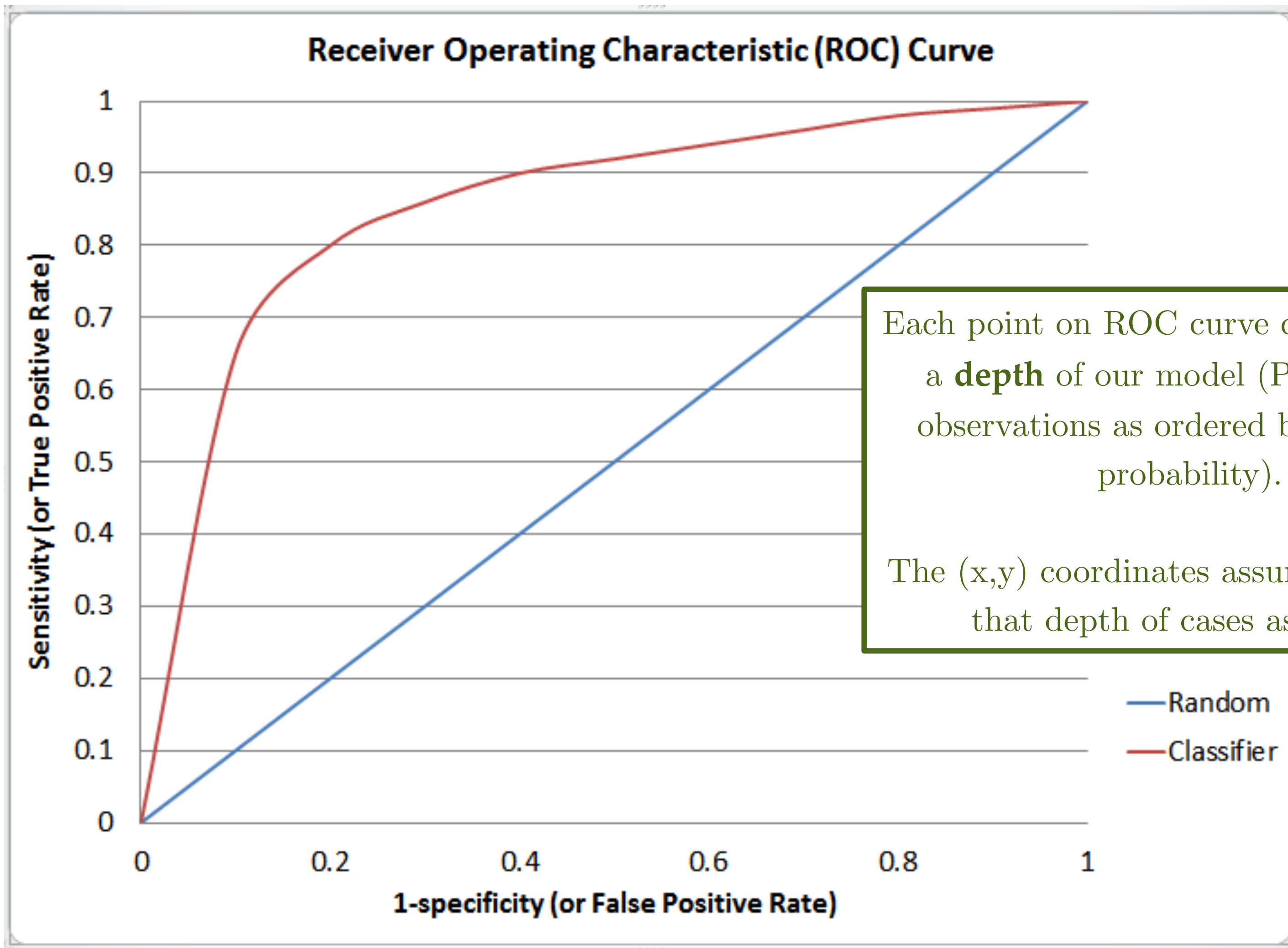
Confusion Matrix

Confusion Matrix		Target			
		Positive	Negative		
Model	Positive	a	b	*Positive Predictive Value*	a/(a+b)
	Negative	c	d	*Negative Predictive Value*	d/(c+d)
		Sensitivity	*Specificity*	**Accuracy** = (a+d)/(a+b+c+d)	
		a/(a+c)	d/(b+d)		

Metrics from Confusion Matrix:

1. Accuracy: Proportion of total predictions that were correct
2. Precision/ Positive Predictive Value: Proportion of predicted positive that were actually positive
3. Negative Predictive Value: Proportion of predicted negative that were actually negative
4. Sensitivity/Recall: Proportion of actual positive cases correctly identified
5. Specificity: Proportion of actual negative cases which are correctly identified

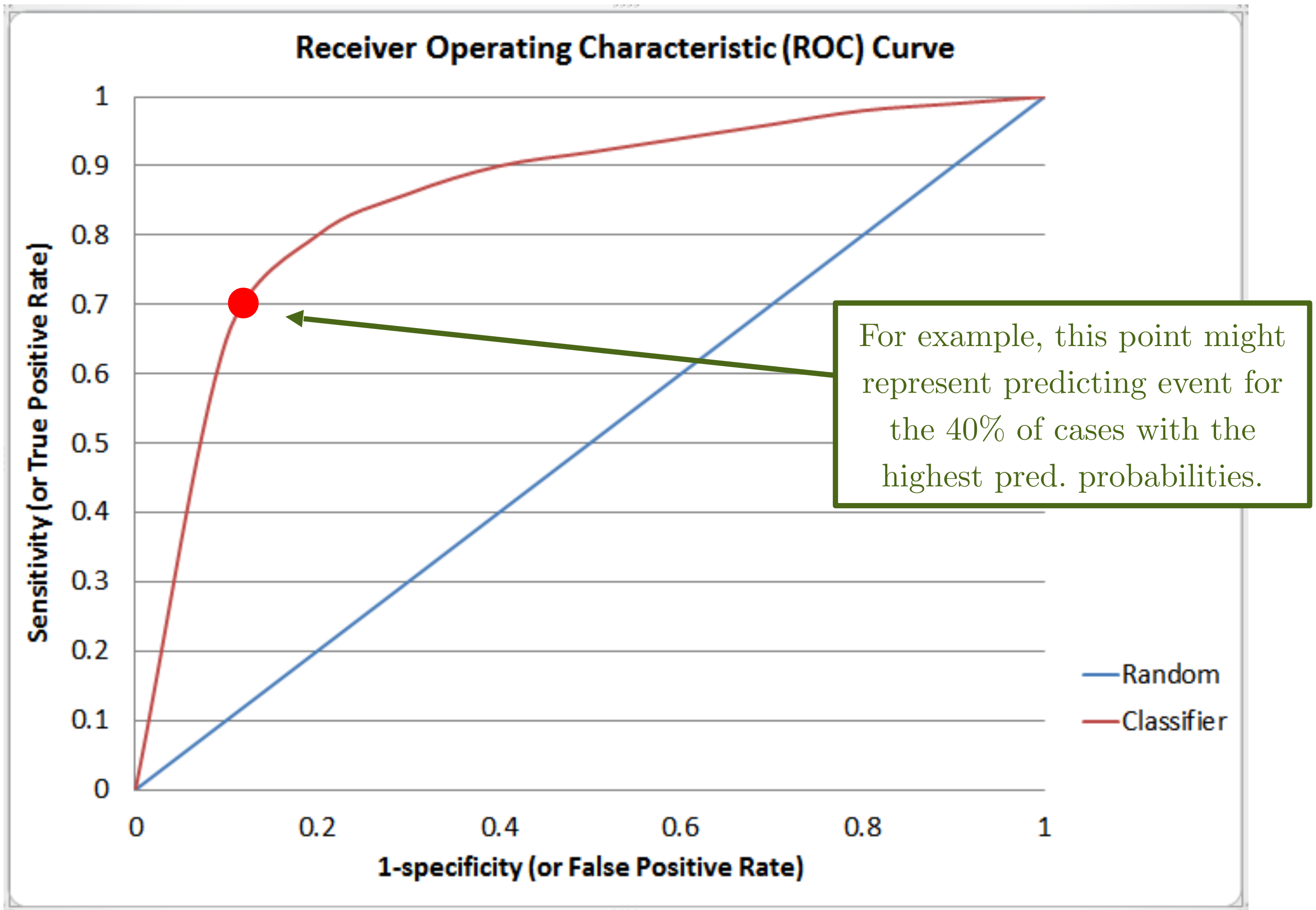
ROC Charts



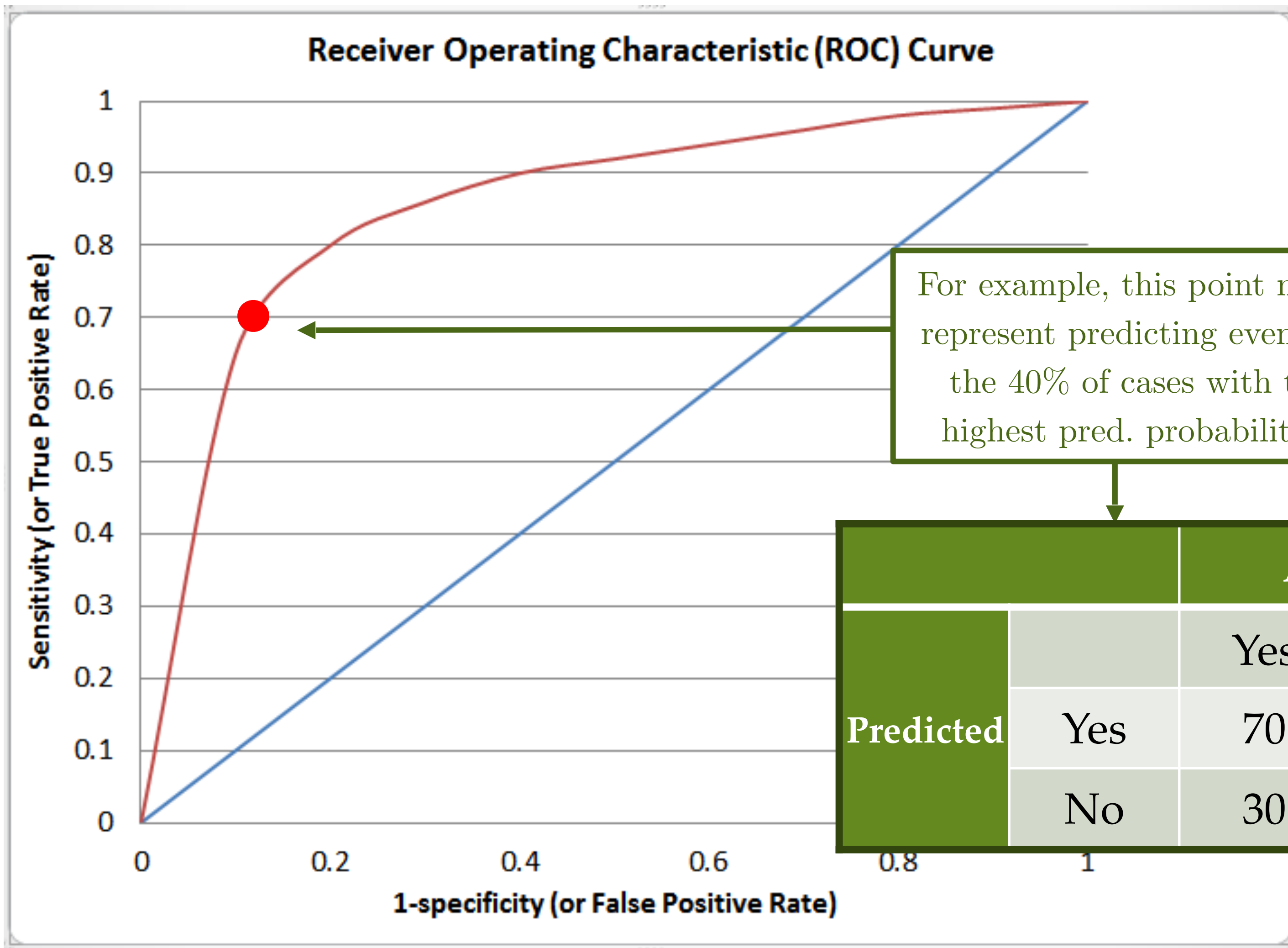
Each point on ROC curve corresponds to a **depth** of our model (Percentile of observations as ordered by predicted probability).

The (x,y) coordinates assume we predict that depth of cases as events.

ROC Charts


Receiver Operating Characteristic (ROC) Curve
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Sensitivity (or True Positive Rate)
0
0.2
0.4
0.6
0.8
1
1-specificity (or False Positive Rate)
Random
Classifier
For example, this point might represent predicting event for the 40% of cases with the highest pred. probabilities.

ROC Charts



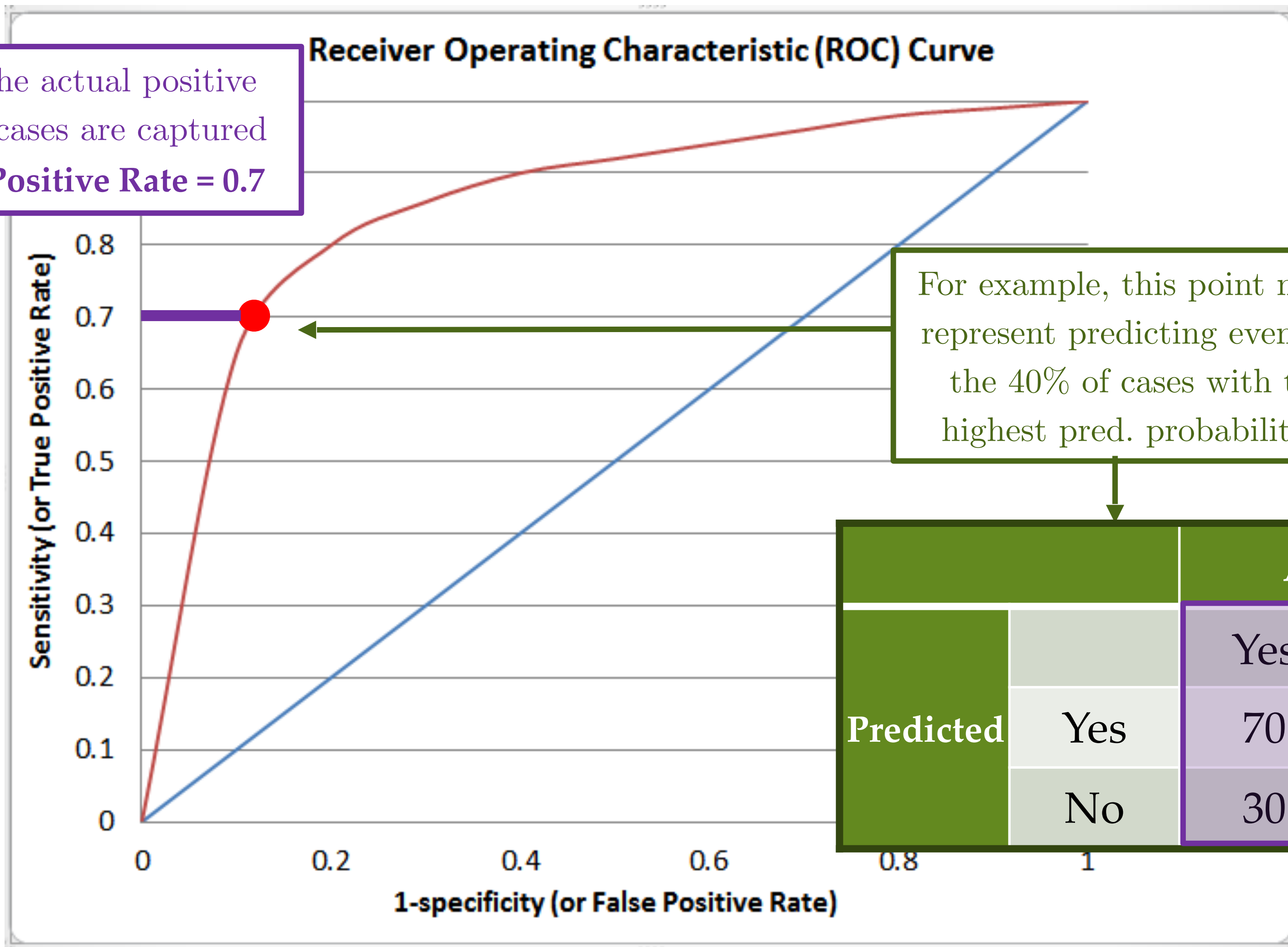
For example, this point might represent predicting event for the 40% of cases with the highest pred. probabilities.

		Actual	
		Yes	No
Predicted	Yes	70	10
	No	30	90

ROC Charts

70% of the actual positive outcome cases are captured

=> True Positive Rate = 0.7



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		Actual	
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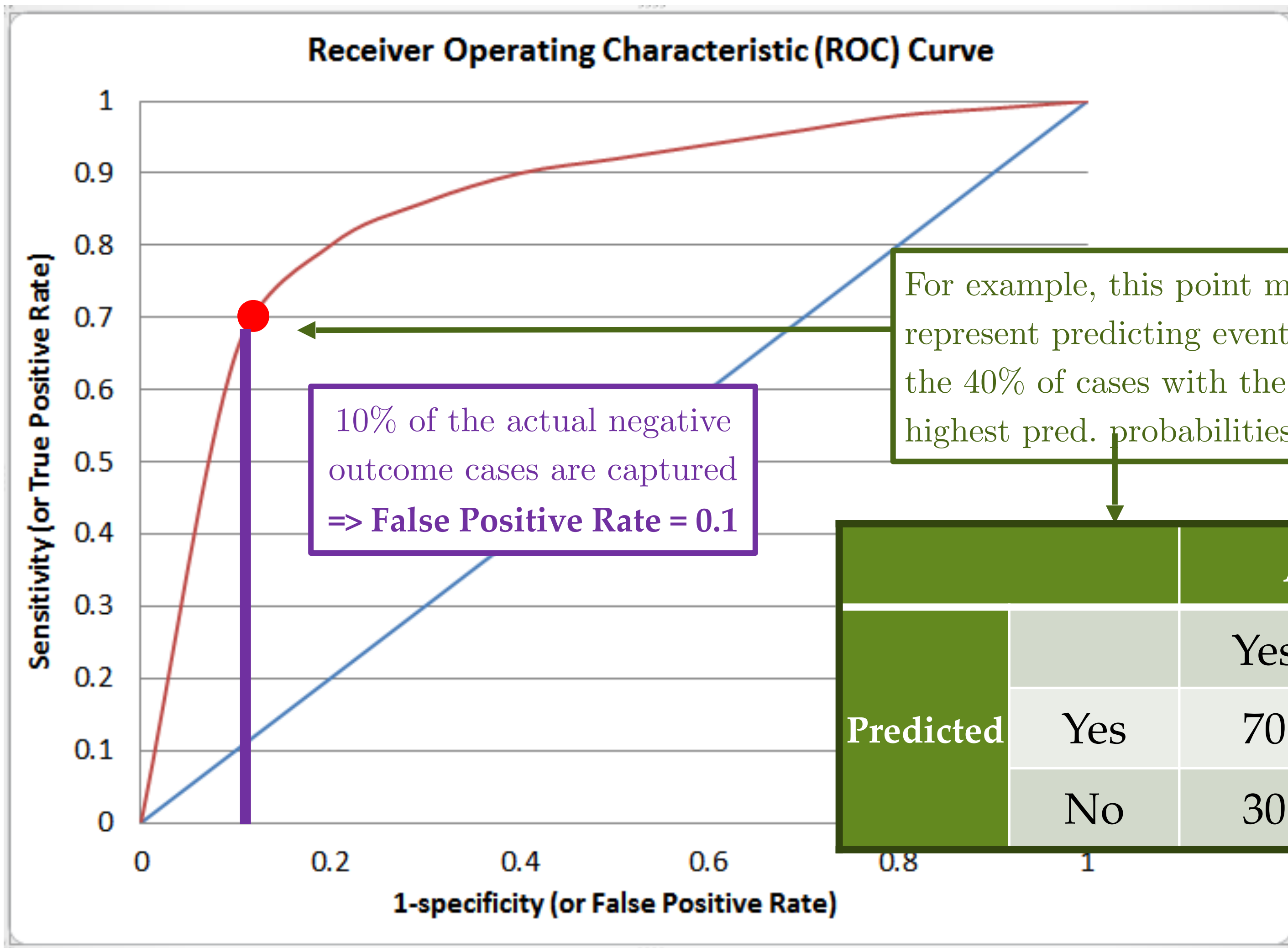
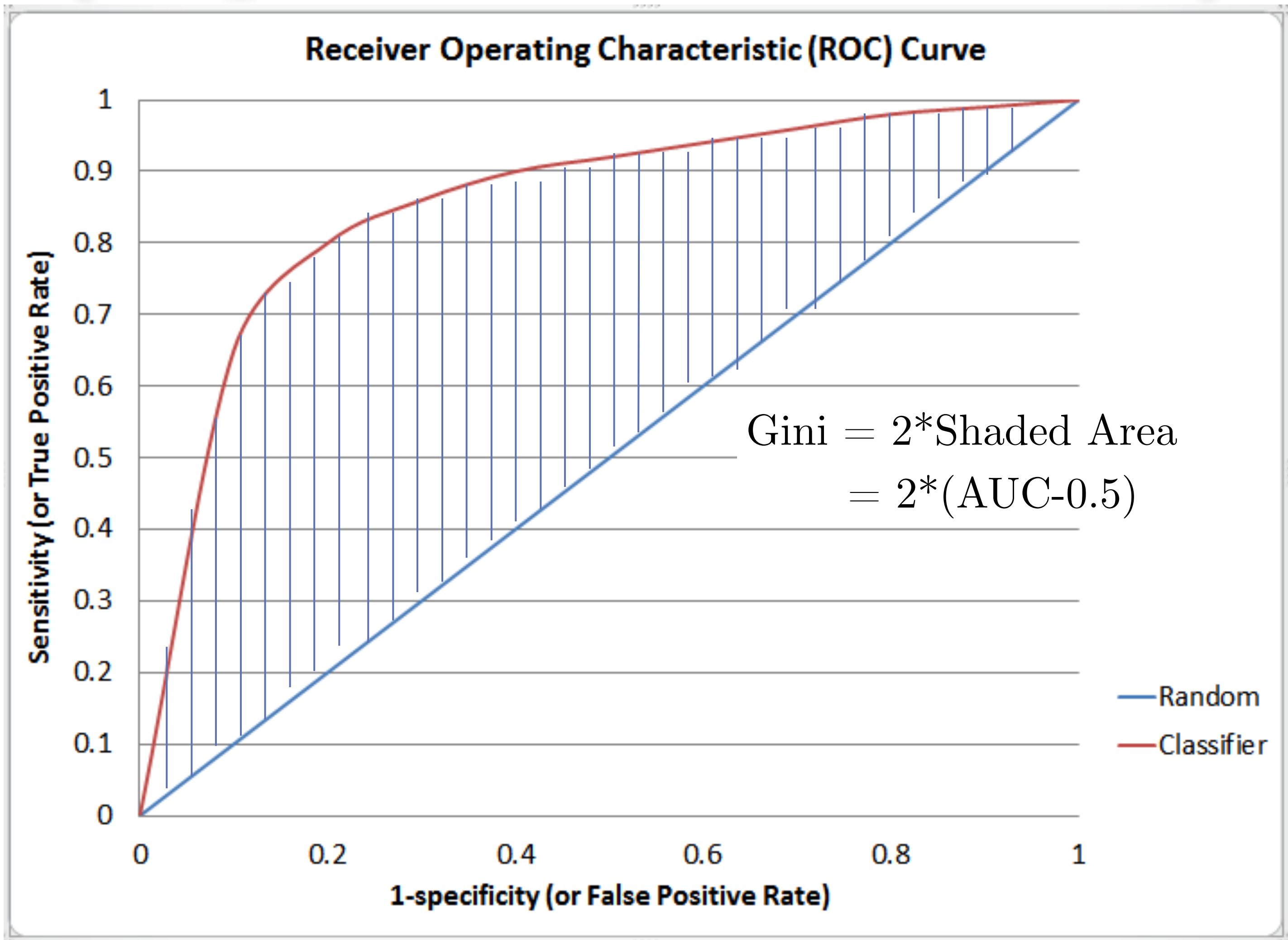
ROC Charts

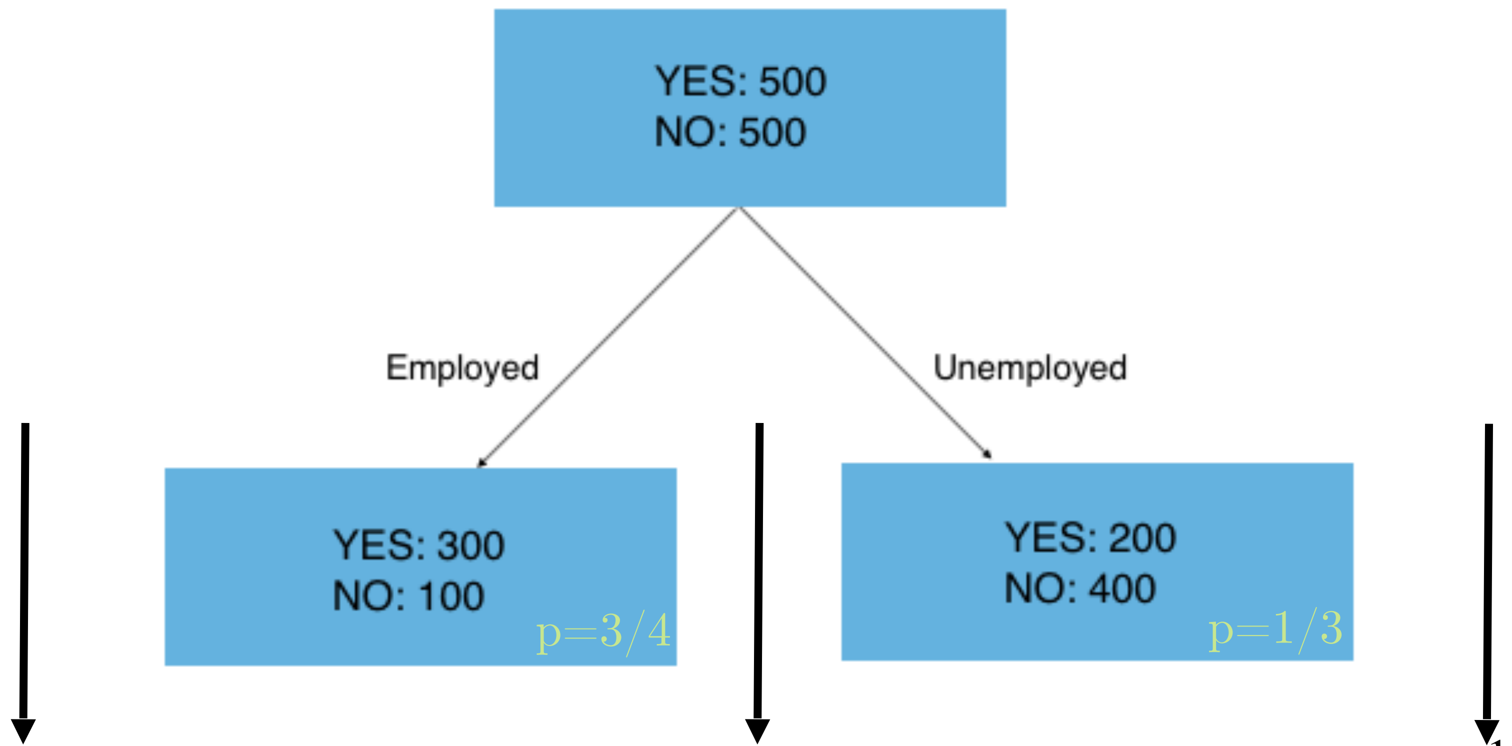


		Actual	
		Yes	No
Predicted	Yes	70	10
	No	30	90

Gini Coefficient (Equivalent to AUC)



ROC Charts for Decision Trees



$p_{cutoff} = 1$

TPR $= 0$

FPR $= 0$

$\frac{1}{3} < p_{cutoff} < \frac{3}{4}$

TPR $= 0.6$

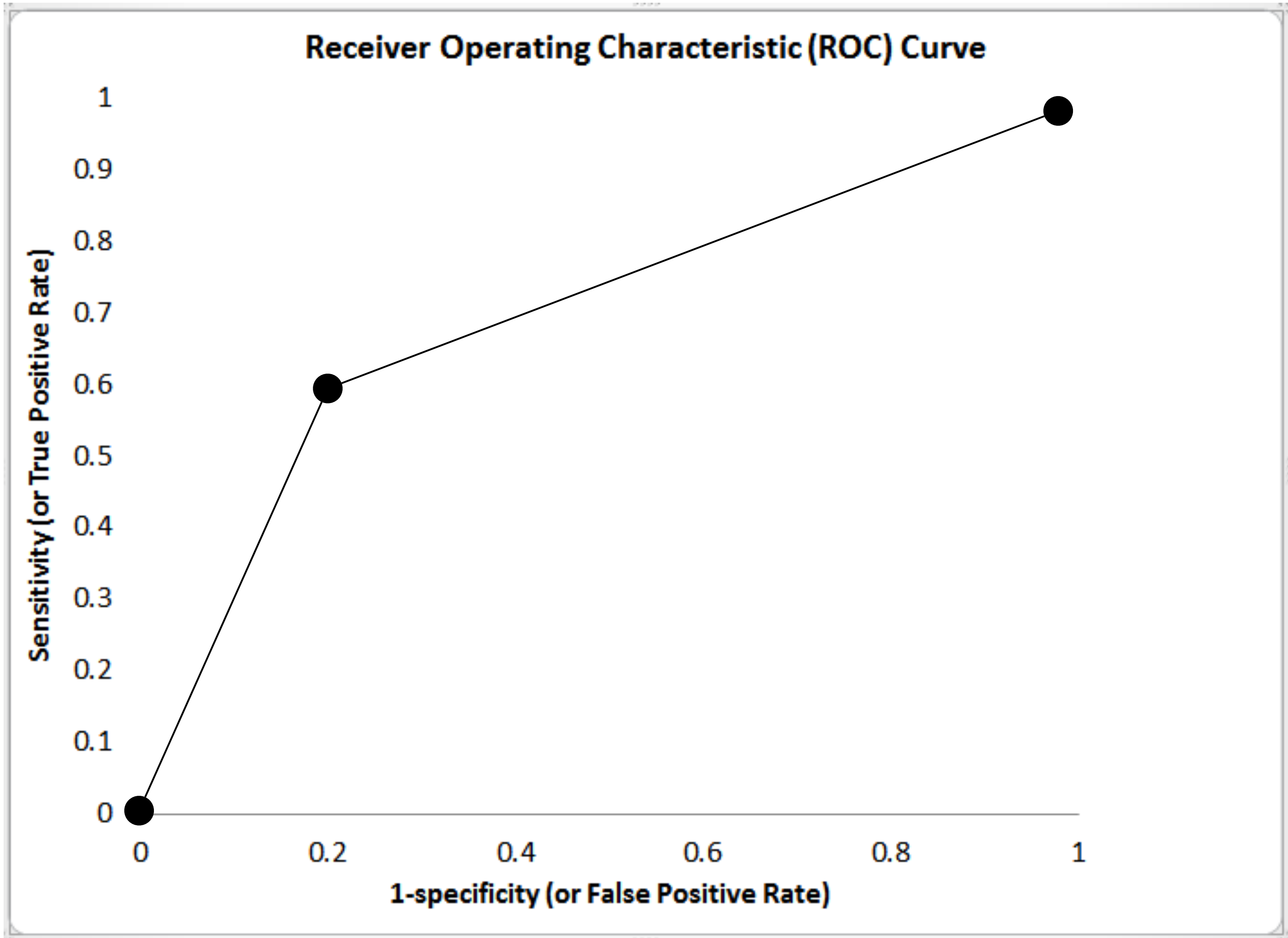
FPR $= 0.2$

$p_{cutoff} < \frac{1}{3}$

TPR $= 1$

FPR $= 1$

ROC Charts for Decision Trees



Average Squared Error (ASE)

$$\frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L}(y_{ij}-\hat{y}_{ij})^2$$

- **Computes sum of squared error between probabilities and binary (0/1) target.**
- For class targets, let L be the number of levels in the target.
- This objective function sets $y_{ij} = 1$ if observation i takes level j of the target and 0 otherwise.

Average Squared Error (ASE)

$$\frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L}(y_{ij}-\hat{y}_{ij})^2$$

Example:

Name	P(red)	P(blue)	P(none)		Actual
JimBob	0.3	0.4	0.3		BLUE
BillyBob	0.1	0.5	0.4		NONE

Average Squared Error (ASE)

$$\frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L}(y_{ij}-\hat{y}_{ij})^2$$

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Name	P(red)	P(blue)	P(none)		Actual
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P(red)	P(blue)	P(none)
0	1	0

Average Squared Error (ASE)

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$$\frac{(0-0.3)^2+(1-0.4)^2+(0-0.3)^2 \quad + \quad (0-0.1)^2+\left(0-0.5\right)^2+(1-0.4)^2}{2*3}$$

Average Squared Error (ASE)

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Things Customers Say

- "We need a model that is accurate when it signals an event is coming - false positives can cause unpredictable losses."

  Lift at Depth

  Positive Predicted Value

- "We need a model that sorts out group A from group B as best as possible."

  Misclassification Rate

  K-S Statistic

- "We need to develop a risk score to measure a client's likelihood of default."

  Log Likelihood

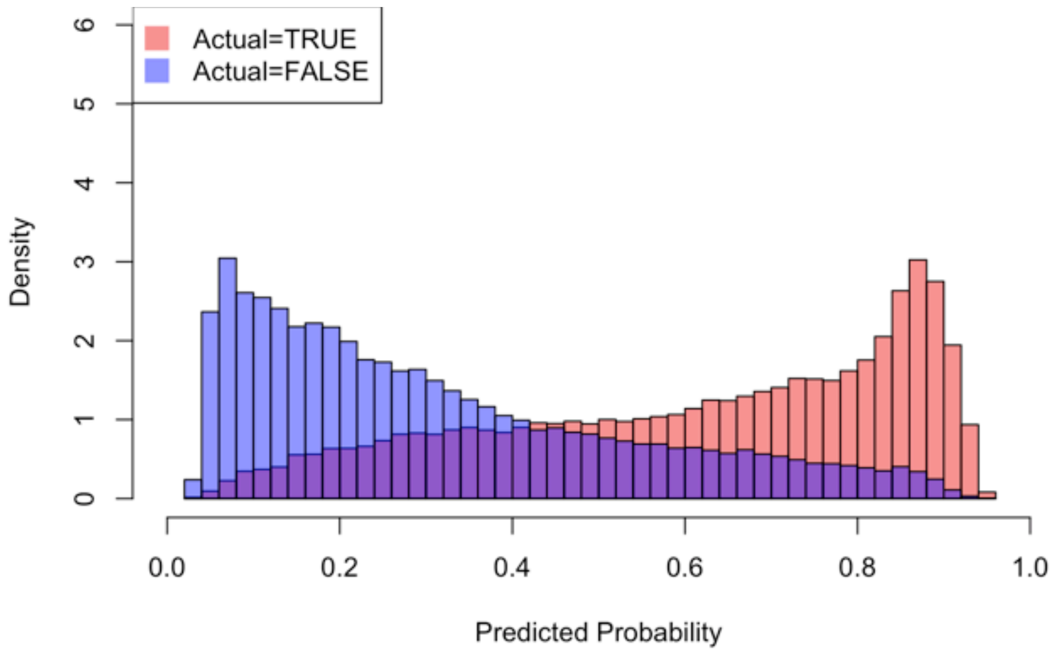
  Average Squared Error

- "We want to rank our machines in terms of failure likelihood so we can rotate through daily maintenance in a logical ordering."

  AUC

  c-statistic

Other Visual Exploration



Plot the distribution of predicted probabilities for each level of the target value.

We'd want these distributions to look as distinct as possible.

Here I used overlaid histogram with transparent colors so you can *see* both distributions.

In case you want to steal my picture:

```
hist(test$pred.probs[test[,"target"]==1], breaks=50, freq=F,   xlim=c(0,1),ylim=c(0,9), col=rgb(1,0,0,0.5),
xlab="Predicted Probability", ylab="Density", main="Test Data Distribution of Predicted Prob. by Actual Outcome" )

hist(test$pred.probs[test[,"target"]==0], breaks=50,freq=F, xlim=c(0,1),ylim=c(0,9), col=rgb(0,0,1,0.5), xlab="",
ylab="Density", add=T)

legend("topleft", legend=c("Actual=TRUE","Actual=FALSE"), col=c(rgb(1,0,0,0.5),rgb(0,0,1,0.5)), pt.cex=2, pch=15 )
```

Undersampling, Oversampling and Prior Probabilities

• • •

How to adjust your model to account for under/oversampling

Undersampling and Prior Probabilities

- Say you have a rare event as target (<10% of data)
  - Fraud
  - Catastrophic failure
  - 10%± single day change in value of stock market index
- May have trouble modeling because a model is accurate for classifying everything as nonevent!
- *Potential* Solution: Create a biased sample

Undersampling and Prior Probabilities

Undersample:

- Under-represent common events in training data.
- Keep all rare events and only a fraction of common events
- Ratio of Common:Rare events is up for debate.
    - 70:30 ought to be fine.
    - 50:50 is sometimes encouraged.

Oversample:

- Replicate the rare events in training.
- Do this *after* the training/validation split so don't have the same observation in both training and validation set!
- OR, use a hybrid technique like **SMOTE** (Chawla, 2002) that creates new data points *like* the rare events (not exact replicates)

Undersampling and Prior Probabilities

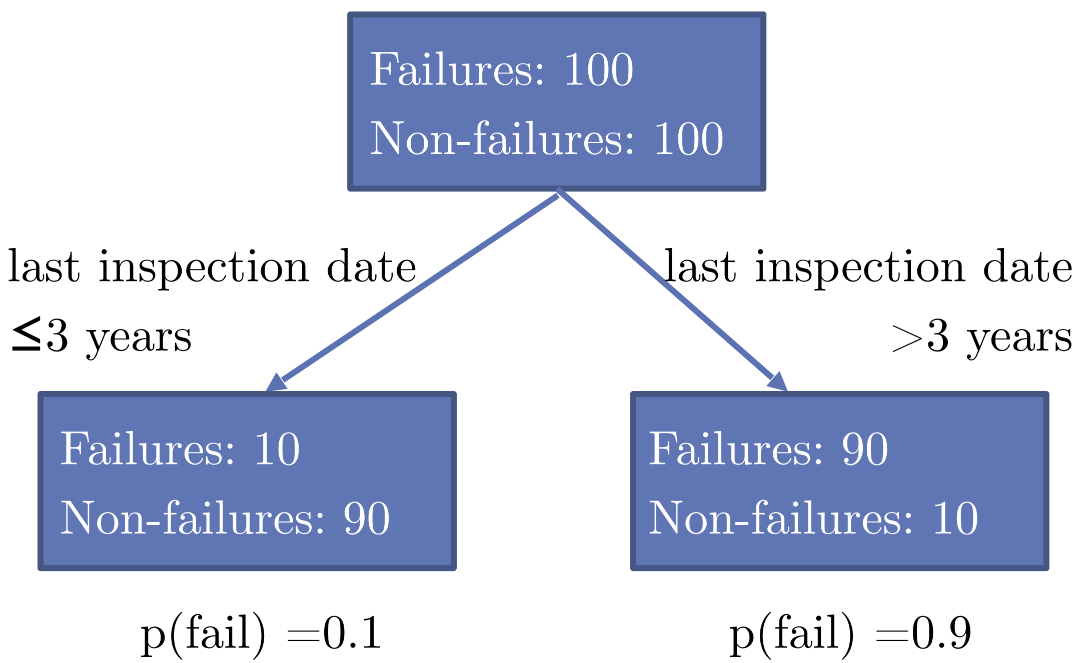
- Models provide **posterior probabilities** for events.
- The accuracy of the posterior probabilities rely on a representative sample.
- If we bias our sample, must adjust the posterior probabilities to account for this.

Why Adjustment is Necessary

Goal: Predict voting machine failure. Only 100 voting machines failed out of 10,000.

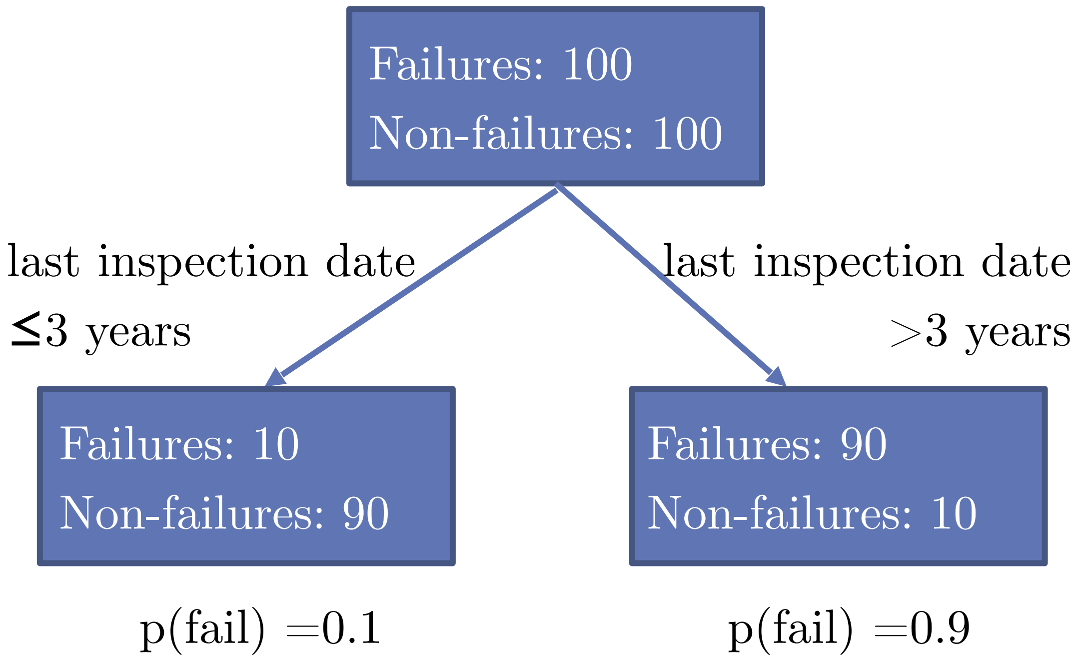
Undersample: Dataset has 100 failures and 100 non-failures.

Create Model:



Why Adjustment is Necessary

Does a new machine with last inspection date >3 years really have a 90% probability of failing?



Why Adjustment is Necessary

- We'd have to go back to the data to answer this question.
- Assuming the 100 non-failures chosen were random, representative sample, we expect inspection date to be ≤ 3 years 90% of the time.
- That is 8,910 non-failing machines with inspection date ≤ 3 years. (8,910 = 90% of 9,900)
- Similarly, 10% of non-failures have expect inspection date >3 years ago. This is 990 machines.

	≤ 3 years	>3 years
Failures	10	90
Nonfailures	8910	990

P(Failure | last inspection date >3 years)
90/(90+990) = 8%
(Still failing at 8 times the rate of recently inspected machines)

Summary: Adjusting for Undersampling

- Let $l = l_1,\ l_2, \ldots,\ l_L$ be the levels of the target variable
- Let $i = 1{,}2, \ldots,\ n$ index the observations in the data
- Let $OldPost(i, l)$ be the posterior probability from the model on oversampled data
- Let $OldPrior(l)$ be the proportion of target level in the oversampled data
- Let $Prior(l)$ be the correct proportion of target level in true population

$$NewPost(i, l) = \frac{OldPost(i, l)\frac{Prior(l)}{OldPrior(l)}}{\sum_{j=1}^{L} OldPost(i, l_j)\frac{Prior(l_j)}{OldPrior(l_j)}}$$