

# Review Jeopardy

Blue vs. Orange

# How this works

- ▶ For the Jeopardy and Double Jeopardy rounds, each question has a dollar amount,  $D$ .
- ▶ Suppose that the proportion of students answering the question correctly is  $p$ .
- ▶ Then the amount earned by the class for that question is  $pD$ .

# Jeopardy Round

Lectures 0-3

# \$200

How could I measure how far apart (i.e. how different) two observations,  $\mathbf{y}_1$  and  $\mathbf{y}_2$ , are from each other?

- (A) Compute  $\|\mathbf{y}_1 - \mathbf{y}_2\|$
- (B) Compute  $\|\mathbf{y}_2 - \mathbf{y}_1\|$
- (C) Compute  $\|\mathbf{y}_1\| - \|\mathbf{y}_2\|$
- (D) Compute  $covariance(\mathbf{y}_1, \mathbf{y}_2)$
- (E) Either (A) or (B)

\$200

What is the span of one vector in  $\mathbb{R}^3$ ?

- (A) A vector
- (B) A line
- (C) A plane
- (D) All of  $\mathbb{R}^3$
- (E) What's  $\mathbb{R}^3$ ?

# \$400

What is the span of two linearly independent vectors in  $\mathbb{R}^3$ ?

- (A) A plane
- (B) A line
- (C) The whole 3-dimensional space
- (D) Orthogonal
- (E) This isn't going well already

# \$400

For three vectors,  $\mathbf{x}$ ,  $\mathbf{y}$  and  $\mathbf{z}$ , suppose that

$$2\mathbf{x} + 5\mathbf{y} + 3\mathbf{z} = \mathbf{0}$$

- (A) Then  $\mathbf{x}$ ,  $\mathbf{y}$  and  $\mathbf{z}$  are linearly independent
- (B) Then  $\mathbf{x}$ ,  $\mathbf{y}$  and  $\mathbf{z}$  are linearly dependent
- (C) Then  $\mathbf{x}$ ,  $\mathbf{y}$  and  $\mathbf{z}$  are orthogonal
- (D) None of the above.
- (E) Both (A) and (B) and sometimes (C). And (D)

# \$600

If a collection of vectors is mutually orthogonal then those vectors are linearly independent.

- (A) True
- (B) False
- (C) Honey Badger



# \$800

If  $\mathbf{U}$  is an orthogonal matrix, then

(A)  $\mathbf{U}^T \mathbf{U} = 0$

(B)  $\mathbf{U}^T \mathbf{U} = \mathbf{U} \mathbf{U}^T = \mathbf{I}$

(C)  $\mathbf{U}$  is a covariance matrix

(D) Both (B) and (C)

(E) I don't know  $\mathbf{U}$ .

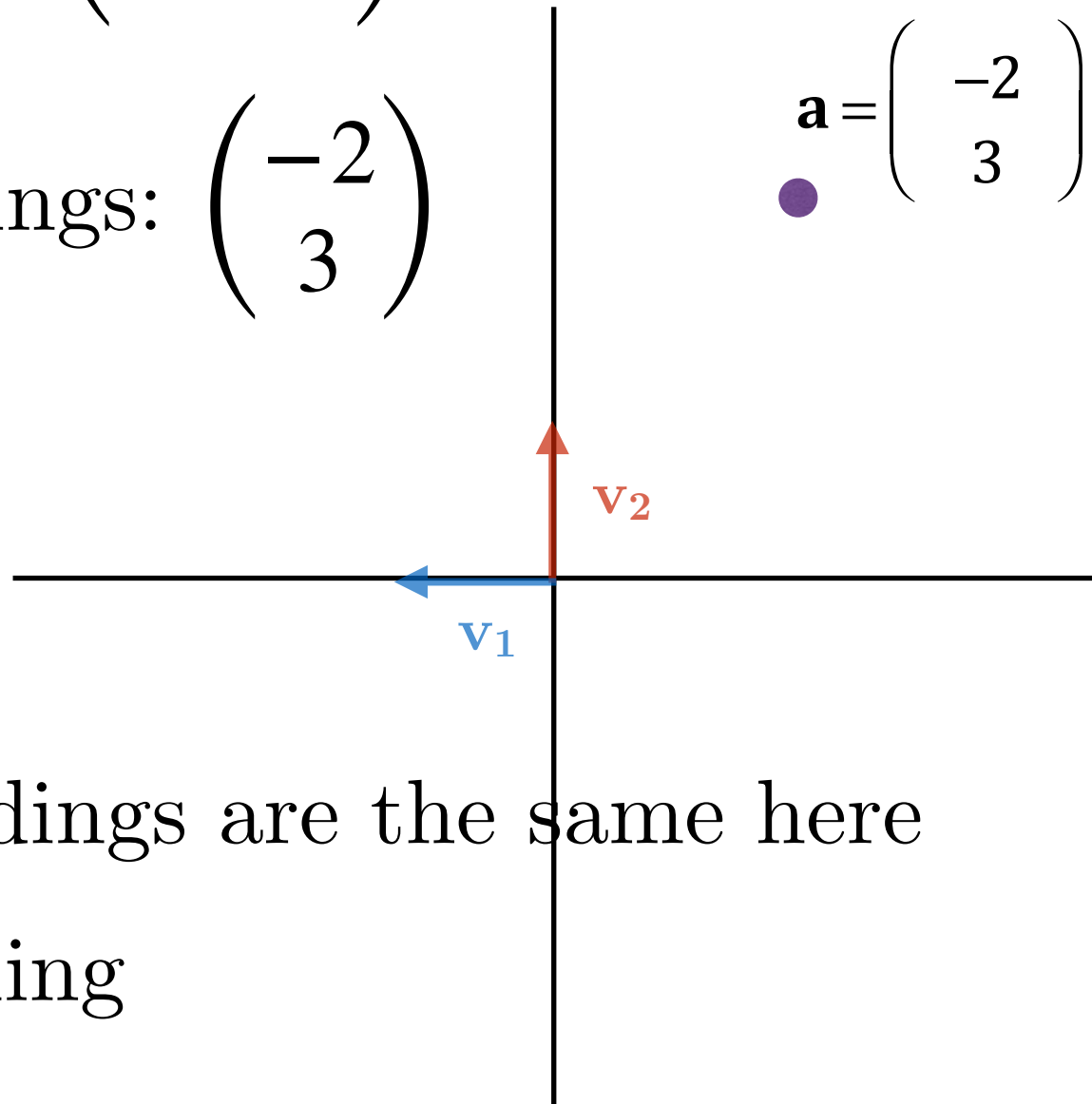
# \$1000

What are the loadings and what are the scores in the picture on the right?

(A) Scores:  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$  Loadings:  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(B) Scores:  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$  Loadings:  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$\mathbf{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} = -2\mathbf{v}_1 + 3\mathbf{v}_2$



(C) The scores and the loadings are the same here

(D) My scores are still loading

# \$1000

If the span of 3 vectors,  $\mathbf{x}$ ,  $\mathbf{y}$  and  $\mathbf{z}$ , is a 2-dimensional subspace (a plane) then

- (A)  $\mathbf{x}$ ,  $\mathbf{y}$  and  $\mathbf{z}$  are multiples of each other
- (B)  $\mathbf{x}$ ,  $\mathbf{y}$  and  $\mathbf{z}$  are linearly independent
- (C)  $\mathbf{x}$ ,  $\mathbf{y}$  and  $\mathbf{z}$  are linearly dependent
- (D)  $\mathbf{x}$ ,  $\mathbf{y}$  and  $\mathbf{z}$  are orthogonal
- (E) \$1K says this is the wrong answer.

# \$1000

In order for a matrix to have eigenvalues and eigenvectors, what must be true?

- (A) All matrices have eigenvalues and eigenvectors
- (B) The matrix must be square
- (C) The matrix must be a covariance matrix
- (D) The matrix must be orthogonal
- (E) The matrix must multiply with an eigenmatrix.

# \$1000

If I multiply a matrix  $\mathbf{A}$  by its eigenvector  $\mathbf{x}$ , what can I say about the result,  $\mathbf{Ax}$ ?

- (A) The result is a unit vector
- (B) The result is a scalar, which is called the eigenvalue
- (C) The result is a scalar multiple of  $\mathbf{x}$
- (D) The result is orthogonal
- (E) Nice try. You can't do that.

# Double Jeopardy Round

Lectures 4-7

# \$400

If your data matrix has 1,000 observations on 40 variables, then how many principal components exist?

(A) 40,000

(B) 1,000

(C) 40

(D) Impossible to know ahead of time

(E) principal. principle? Principal.

# \$400

The first principal component is...

- (A) A statistic that tells you how much multicollinearity is in your data
- (B) A scalar that tells you how much total variance is in your data
- (C) The first column of your data matrix
- (D) A vector that points in the direction of maximal variance of your data
- (E) The lego piece that is *all* the way under the couch



# \$800

The loadings on a principal component tell you...

- (A) The variance of each variable on that component
- (B) The relative influence/weight of each variable on that component
- (C) The coordinates of the observations after projection onto that component
- (D) The efficiency of the calculation of that component
- (E) Not a whole lot.

# \$1200

The principal component scores are...

- (A) Statistics which tell you the relative importance of each principal component
- (B) The coordinates of your data in the new basis of principal components
- (C) Statistics which tell you how each variable relates to each component
- (D) Rarely used for analysis
- (E) Higher than my fantasy football scores.

# \$1200

The eigenvalues of the covariance matrix...

- (A) Are always orthogonal
- (B) Add up to 1
- (C) Tell you how much variance exists along each principal component
- (D) Tell you the proportion of variance explained by each principal component
- (E) Are the same as the covalues of the eigenvariance matrix.

# \$1600

The total amount of variance in a dataset is...

- (A) The sum of all entries in the covariance matrix
- (B) The sum of the eigenvalues
- (C) The sum of the variances of each variable in data
- (D) Both (B) and (C)

(E) 
$$\sum_{i=1}^n \sigma_i^2 \frac{\sqrt{\lambda_i}}{\frac{1}{2} \mathbf{u}^T \mathbf{v}} + J(\mathbf{X}^T \mathbf{X})$$

# \$1600

PCA is a special case of the Singular Value Decomposition (SVD), when your data matrix is either centered or standardized.

- (A) True
- (B) False
- (C) Woodpecker

# \$1600

Principal components regression...

- (A) Is a biased regression technique
- (B) Can be used to solve the problem of severe multicollinearity as long as you omit some principal components
- (C) Can give you parameter estimates in terms of your dataset's original variables
- (D) All of the above
- (E) False.

# \$2000

When we perform a rotation of principal components, we may not explain as much total variance as we did before rotation but we will always explain more variance on the first component after rotation.

(A) True

(B) False

(C) Is this related to the lost lego?

# Final Jeopardy

Category: PCA Rotations



# How this works.

- ▶ This is the only place you can lose money
- ▶ You'll select a wager,  $W$
- ▶ Some proportion,  $p$ , of you will get the question right.

IF  $p > 0.80$  then Payout =  $pW$

ELSE Payout =  $-(1-p)W$

# Choose your Wager

- (A) 0
- (B) 2000
- (C) 4000
- (D) 5000
- (E) 8000

# Final Jeopardy Question

What is the purpose or motivation behind the rotations of principal components in Factor Analysis?

- (A) The original principal components were not orthogonal so we need to adjust them
- (B) The first principal component does not explain enough variance. By rotating we can explain more variance.
- (C) The loadings of the variables are difficult to interpret, by rotating we get new factors that more clearly represent combinations of original variables.
- (D) The rotation helps spread out the observations so we can more clearly see differences between groups or classes in the data.

