

Principal Components Analysis - Worksheet

Part One

1. Suppose we have a dataset with 8 variables and we use *standardized* data (i.e. correlation PCA). What is the total amount of variance in our data?

8

2. Suppose I have a dataset with 3 variables and the eigenvalues of the covariance matrix are

$$\lambda_1 = 3, \lambda_2 = 2, \lambda_3 = 1.$$

- a. What proportion of variance is explained by the first principal component?

$\frac{1}{2}$

- b. What is the variance of the second principal component?

2

- c. What proportion of variance is captured by using both the first and second principal components?

$\frac{5}{6}$

3. The following output is produced in SAS after running PCA on the iris dataset:

Eigenvalues of the Correlation Matrix				
	Eigenvalue	Difference	Proportion	Cumulative
1	2.91849782	2.00446735	0.7296	0.7296
2	0.91403047	0.76727360	0.2285	0.9581
3	0.14675688	0.12604204	0.0367	0.9948
4	0.02071484		0.0052	1.0000

Eigenvectors				
	Prin1	Prin2	Prin3	Prin4
Sepal_Length	0.521066	0.377418	-.719566	-.261288
Sepal_Width	-.269347	0.923296	0.244382	0.123510
Petal_Length	0.580413	0.024492	0.142126	0.801449
Petal_Width	0.564857	0.066942	0.634273	-.523597

- How much variance in the data is captured by a projection onto the span of the first three principal components?

99%

- Which variable is most closely associated with PC 2?

sepal width

- Observations that have larger scores on PC3 are somewhat likely to have larger/smaller than average sepal lengths? (circle one)

- If you had to reduce the dimensions of this data down to 2 variables, which variables would you choose?

principal component 1 and principal component 2

- What is the total amount of variance for this example? How do you know?

4. The top box says eigenvalues of correlation matrix so this is correlation PCA and there are 4 variables in our data.

List of Key Words/Phrases.

eigenvalue

eigenvector

principal components

directional variance

proportion of variance

correlation vs covariance PCA

orthogonal projection onto PCs

PCA loadings

PCA coordinates

biplot

zero eigenvalues

small eigenvalues