Singular Value Decomposition - Worksheet

Give an explanation for True/False

1. *(True/**False**.)* The matrix $\mathbf{D}$ in the singular value decomposition is diagonal and contains the same eigenvalues that are output in PCA.
   It contains the square roots of those eigenvalues, which are called singular values.

2. *(**True**/False.)* Principal Component Analysis involves the SVD of a data matrix which has been centered or standardized.

3. *(**True**/False.)* The matrices $\mathbf{U}$ and $\mathbf{V^T}$ in the singular value decomposition are orthogonal, and that means their inverse is equal to their transpose.

4. *(True/**False**.)* When we omit principal components with small eigenvalues, the information we lose has a larger signal-to-noise ratio than the information we keep.
   smaller signal-to-noise ratio. They are mostly noise.

BONUS: *(**True**/False.)* The default factors displayed in SAS's proc factor results window are the rows of the matrix product $\mathbf{DV^T}$ where $\mathbf{X} = \mathbf{UDV^T}$ is the SVD of your standardized data matrix $\mathbf{X}$.

Facts:

(a) proc factor uses correlation PCA by default

(b) the factors that are displayed in the output are the principal components (rows of $\mathbf{V^T}$) only each is scaled by the square root of it's eigenvalue.

(c) the matrix $\mathbf{D}$ contains the square roots of those eigenvalues

(d) multiplication by a diagonal matrix on the left will scale the rows of a matrix by the corresponding diagonal elements.

(e) $\longrightarrow$ $\square$

List of Key Words/Phrases.

singular value decomposition   signal-to-noise ratio
PCA and SVD relationship