Orthogonality - Worksheet

1. What is the cosine of the angle between $\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$?

2. Are vectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ orthogonal? How do you know?

3. What are the two conditions necessary for a collection of vectors to be orthonormal?

4. Briefly explain why an orthonormal basis is important.

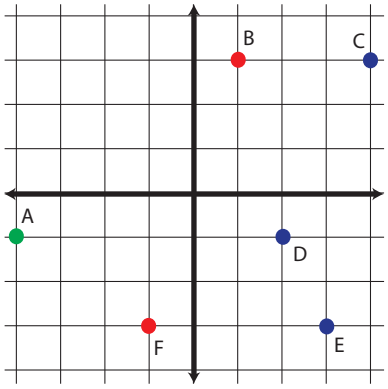
1. Let $\mathbf{U} = \frac{1}{3} \begin{pmatrix} -1 & 2 & 0 & -2 \\ 2 & 2 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ -2 & 1 & 0 & 2 \end{pmatrix}$

   a. Show that $\mathbf{U}$ is an orthogonal matrix.

   b. Let $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$. Solve the equation $\mathbf{U}\mathbf{x} = \mathbf{b}$.

2. Find two vectors which are orthogonal to $\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

3. Draw the orthogonal projection of the points onto the subspace $span \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$



List of Key Words.

cosine
orthogonal
orthonormal
orthogonal Matrix
orthonormal basis
orthogonal projection