Challenge Puzzle

Hints toward Solution

Challenge Puzzle

- Suppose we have 1,000 individuals that have been divided into 5 different groups each year for 20 years.
- We need to make a 1000x1000 matrix **C** where

$$\mathbf{C_{ij}} = \#\text{ times person } i \text{ grouped with person } j$$

- The data currently has 1000 rows and 5x20 = 100 binary columns indicating whether each individual was a member of each group (yLgK: yearLgroupK): (y1g1, y1g2, y1g3, y1g4, y1g5, y2g1, ... y20g5)
- Can we use what we've just learned to help us here?

Hint 1

Define a subgoal

Hint 1

Think about an "adjacency matrix" for each year. For each year, this matrix would be binary and indicate whether or not the individuals were in the same group.

$$\mathbf{A_{ij}} = 1 \text{ if person } i \text{ grouped with person } j,$$
$$\mathbf{A_{ij}} = 0 \text{ otherwise}$$

If you had such a matrix for each year, you could simply add them up to get the desired matrix **C**

Hint 2

A piece of the subgoal

Hint 2

Suppose I have 4 people and I divide them into group A and group B:

Person	Group A	Group B
Jojo	1	0
Mimi	1	0
Lily	0	1
Mr. T	0	1

Let's look at the outer product of the first column with itself:

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Hint 2

If that doesn't quite help, let's label the columns:

$$\begin{matrix} \text{Jojo} \\ \text{Mimi} \\ \text{Lily} \\ \text{Mr. T} \end{matrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \text{Jojo} & \text{Mimi} & \text{Lily} & \text{Mr. T} \\ 1 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \text{Jojo} & \text{Mimi} & \text{Lily} & \text{Mr. T} \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \\ \text{Jojo} \\ \text{Mimi} \\ \text{Lily} \\ \text{Mr. T} \end{matrix}$$

This product gives us *part* of the adjacency matrix for that one year...the part for group A...

Hint 3

Finding the subgoal solution. Sort of.

Hint 3

To get the *whole* adjacency matrix for that year, we can add in the part for group B:

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Hint 4

Thiiiinnnkkkk about it

Hint 4

The best part is when we factor in the information from this next slide of the notes:

Matrix-Matrix Multiplication (O-P View)

We can write the product $\mathbf{AB}$ as a *sum of outer products* of columns of $\mathbf{A}_{(\mathbf{m}\times\mathbf{n})}$ and rows of $\mathbf{B}_{(\mathbf{n}\times\mathbf{p})}$

$$\mathbf{AB} = \sum_{i=1}^{n} \mathbf{A}_{\star i}\mathbf{B}_{i\star}$$

This view decomposes the product $\mathbf{AB}$ into the sum of n matrices, each of which has rank 1 (discussed later).

Hint 5

This is it, right here...

Matrix-Matrix Multiplication (O-P View)

We can write the product $\mathbf{AB}$ as a *sum of outer products* of columns of $\mathbf{A}_{(\mathbf{mxn})}$ and rows of $\mathbf{B}_{(\mathbf{nxp})}$

We can turn it into a matrix product!

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathbf{AB} = \sum_{i=1}^{n} \mathbf{A}_{\star i}\mathbf{B}_{i\star}$$

We have a sum of outer products

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 & 1 \end{pmatrix}$$

This view decomposes the product $\mathbf{AB}$ into the sum of n matrices, each of which has rank 1 (discussed later).

Another View

via InnerProducts

An Easier(!?) Way...

- You might argue that the following is a more straightforward way to view the solution.
- It may be - but the former solution opens the door to other problems that are much more subtle in nature than the straight inner product solution.

The Inner Product

- Let $\mathbf{x}_i{}^{\mathrm{T}}$ be a row of the data matrix as described, so that $\mathbf{x}_i{}^{\mathrm{T}}$ has 100 entries, each specifying whether or not person i was placed in each of the 100 groups.

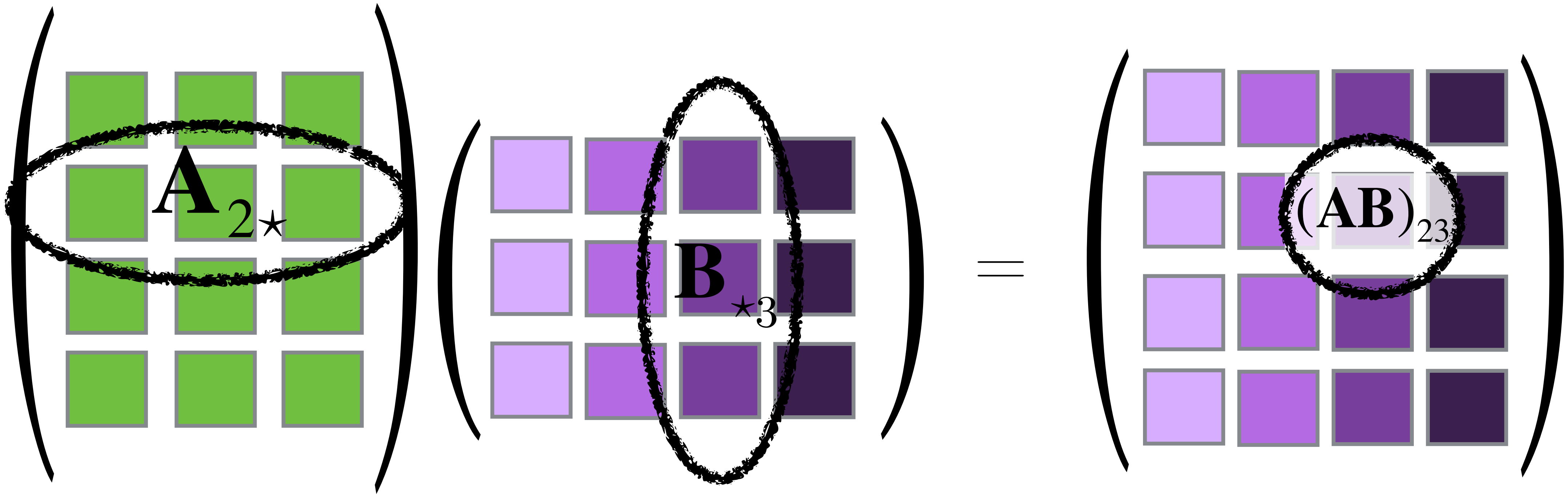
- It should be clear that $\text{sum}(\mathbf{x}_i{}^{\mathrm{T}}) = 20$ for each person i.

- Consider $\mathbf{x}_i{}^{\mathrm{T}}\ \mathbf{x}_j$. What would be the result of this calculation? If you can't see immediately, write an example similar to what we did in the previous solution.

The Inner Product

Now let's use the information from the following slide:

Matrix-Matrix Multiplication (I-P View)



$$(\mathbf{AB})_{ij} = \mathbf{A}_{i\star}\mathbf{B}_{\star j}$$

The Inner Product

So with this in mind, can you figure out how to make the matrix of inner products that provides the solution that we want?