Worksheet - Lecture 12
Linear Independence

1. Determine whether or not the vectors

$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

   are linearly independent. Justify your answer using the definition of linear dependence/independence.

2. What is the rank of the matrix $\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 2 & 0 & 2 \end{pmatrix}$?

   b. How many linearly independent rows does $\mathbf{M}$ have? How many linearly independent columns?

   c. Is $\mathbf{M}$ full rank?

3. What is the difference between *perfect* multicollinearity and *severe* multicollinearity? What are the problems associated with each of these?

4. *(True/False)* If $\mathbf{Ax} = \mathbf{b}$ has a solution then $\mathbf{b}$ can be written as a linear combination of the columns of $\mathbf{A}$.