Worksheet - Lecture 13
Advanced Matrix Arithmetic

1. *(True/False)* If $\mathbf{A} = \mathbf{A}^T$ then $\mathbf{A} = \mathbf{I}$, the identity matrix.

2. *(True/False)* The transpose of a lower triangular matrix is an upper triangular matrix.

3. Simplify the following matrix equations, if possible:
   (Hint: Because of the distributive law, multiplying binomials works the same with matrices as it does with scalars, only the order of the multiplications must be preserved:

   $$(A + B)(A + B) = A^2 + BA + AB + B^2$$

   Also, in case it is not immediately clear to you at this point, we can combine like terms as usual,

   $$AB + AB = 2AB$$

   a. $\mathbf{A}(\mathbf{BC} - \mathbf{CD}) + \mathbf{A}(\mathbf{C} - \mathbf{B})\mathbf{D} - \mathbf{AB}(\mathbf{C} - \mathbf{D})$

   b. $(\mathbf{A} - \mathbf{B})(\mathbf{C} - \mathbf{A}) + (\mathbf{C} - \mathbf{B})(\mathbf{A} - \mathbf{C}) + (\mathbf{C} - \mathbf{A})^2$

   c. $(\mathbf{A}^T\mathbf{C}^T)^T(\mathbf{CA}^T)^T(\mathbf{AC}^T)^T$

   d. $(\mathbf{I} - \mathbf{BA})(\mathbf{I} - \mathbf{BA}) + \mathbf{B}(2\mathbf{A} - \mathbf{ABA})$

   e. $\mathbf{A}^{-1}(\mathbf{B}^2\mathbf{A}^T)^T\mathbf{B}^{-T}$