Worksheet - Lecture 15
Norms and Distance Measures

1. Let $\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ -4 \\ -2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$.

   a. Determine the Euclidean distance between $\mathbf{u}$ and $\mathbf{v}$.

   b. Find a vector of unit length in the direction of $\mathbf{u}$.

   c. Find the 1- and ∞-norms of $\mathbf{u}$ and $\mathbf{v}$.

   d. Find the Manhattan distance between $\mathbf{u}$ and $\mathbf{v}$.

2. What is the 2-norm of a unit vector?

3. Describe in words what happens when you take the inner product of a vector with itself, $\mathbf{x}^T\mathbf{x}$. How does this computation relate to the 2-norm of $\mathbf{x}$, $\|\mathbf{x}\|_2$?

4. (*True/False*) When we have a system of equations for regression analysis,

   $$\mathbf{X}\boldsymbol{\beta} = \mathbf{y}$$

   which has no exact solutions, the goal of the Least-Squares method is to find a solution $\widehat{\boldsymbol{\beta}}$ such that

   $$\|\mathbf{X}\widehat{\boldsymbol{\beta}} - \mathbf{y}\|_2^2$$

   is minimized.
   (In case the notation looks confusing, that is the two-norm squared, $\|\mathbf{x}\|_2^2 = (\|\mathbf{x}\|_2)^2$.)