Worksheet - Lecture 4

Matrix Arithmetic Part Two

1. Use the following matrices to answer the questions:

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 8 \\ 3 & 0 & -2 \\ 8 & -2 & -3 \end{pmatrix} \quad \mathbf{M} = \begin{pmatrix} 1 & 8 & -2 & 5 \\ 2 & 8 & 1 & 7 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\mathbf{H} = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \quad \mathbf{W} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{pmatrix}$$

a. Circle the matrix products that are possible and specify their resulting dimensions:

$\mathbf{AM}$	$\mathbf{W}^T\mathbf{D}$	$\mathbf{M}^T\mathbf{H}^T$
$\mathbf{AW}$	$\mathbf{HM}$	
$\mathbf{WD}$	$\mathbf{MH}$	$\mathbf{DW}$

- Compute the following matrix products:

$$\mathbf{HM} \quad \text{and} \quad \mathbf{AD}$$

- From the previous computation, **AD**, do you notice anything interesting about multiplying a matrix by a diagonal matrix on the right? Can you generalize what happens in words?

Different Views of Matrix Multiplication

2. Consider the matrix product $\mathbf{AB}$ where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

Let $\mathbf{C} = \mathbf{AB}$.

- Compute the matrix product $\mathbf{C}$.

- Compute the matrix-vector product $\mathbf{AB}_{\star 1}$ and show that this is the first column of $\mathbf{C}$. (Likewise, $\mathbf{AB}_{\star 2}$ is the second column of $\mathbf{C}$.) (*Matrix multiplication can be viewed as a collection of matrix-vector products.*)

- Compute the two outer products using columns of $\mathbf{A}$ and rows of $\mathbf{B}$ and show that

$$\mathbf{A}_{\star 1}\mathbf{B}_{1\star} + \mathbf{A}_{\star 2}\mathbf{B}_{2\star} = \mathbf{C}$$

  (*Matrix multiplication can be viewed as the sum of outer products.*)

- Since $\mathbf{AB}_{\star 1}$ is the first column of $\mathbf{C}$, show how $\mathbf{C}_{\star 1}$ can be written as a linear combination of columns of $\mathbf{A}$. (*Matrix multiplication can be viewed as a collection of linear combinations of columns of the first matrix.*)

- Finally, note that $\mathbf{A}_{1\star}\mathbf{B}$ will give the first row of $\mathbf{C}$. (*This amounts to a linear combination of rows - can you see that?*)