

Worksheet - Lecture 11

Least Squares

For the following problems you may use software or an online matrix inverse calculator to compute any matrix inverse required. One option can be found here:

<http://matrix.reshish.com/inverse.php>

- Find the least squares solution to following inconsistent system:

$$\underbrace{\begin{pmatrix} 2 & -2 \\ -2 & 2 \\ 5 & 3 \end{pmatrix}}_X \underbrace{\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}}_{\beta} = \underbrace{\begin{pmatrix} -1 \\ 7 \\ -26 \end{pmatrix}}_y$$

$$X^T X = \begin{pmatrix} 33 & 7 \\ 7 & 17 \end{pmatrix}$$

$$(X^T X)^{-1} = \begin{pmatrix} 0.0332 & -0.0137 \\ 0.0137 & 0.0645 \end{pmatrix}$$

$$X^T y = \begin{pmatrix} -146 \\ -62 \end{pmatrix}$$

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

$$= \begin{pmatrix} 0.0332 & -0.0137 \\ -0.0137 & 0.0645 \end{pmatrix} \begin{pmatrix} -146 \\ -62 \end{pmatrix}$$

$$= \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

- Use the method of least squares to find the regression equation associated with Problem 3 on Worksheet 5 (regarding SAT scores).

$$\hat{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 91.102 \\ 0.717 \\ 0.260 \end{pmatrix}$$

your answers may
be slightly different
depending on how
you round them