Worksheet - Lecture 12
Linear Independence

1. Determine whether or not the vectors

$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

are linearly independent. Justify your answer using the definition of linear dependence/independence.

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 1 & 0 & 2 & 0 \\ 3 & 1 & -1 & 0 \end{array}\right) \xrightarrow[\text{Jordan}]{\text{Gauss -}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

Since the equation $\alpha_1 \mathbf{x}_1 + \alpha_2 \mathbf{x}_2 + \alpha_3 \mathbf{x}_3 = \mathbf{0}$ has only the trivial solution, $\alpha_1 = \alpha_2 = \alpha_3 = 0$, the vectors ARE linearly independent.

2. What is the rank of the matrix $\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 2 & 0 & 2 \end{pmatrix}$?

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 2 & 0 & 2 \end{pmatrix} \xrightarrow{\text{Gauss - Jordan}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

Rank (M) = 2

b. How many linearly independent rows does **M** have? How many linearly independent columns? 2, 2

c. Is **M** full rank? no, rank < 3

3. *(True/False)* If $\mathbf{Ax} = \mathbf{b}$ has a solution then $\mathbf{b}$ can be written as a linear combination of the columns of $\mathbf{A}$.

True!!!