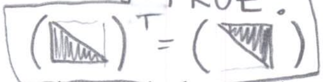
Worksheet - Lecture 13

Advanced Matrix Arithmetic

1. *(True/False)* If $\mathbf{A} = \mathbf{A}^T$ then $\mathbf{A} = \mathbf{I}$, the identity matrix. $\boxed{\text{FALSE. A IS ANY SYMMETRIC MATRIX!!}}$

2. *(True/False)* The transpose of a lower triangular matrix is an upper triangular matrix. TRUE!



3. Simplify the following matrix equations, if possible:
(*Hint: Because of the distributive law, multiplying binomials works the same with matrices as it does with scalars, only the order of the multiplications must be preserved:*

$$(\boldsymbol{A}+\boldsymbol{B})(\boldsymbol{A}+\boldsymbol{B}) = \boldsymbol{A}^2 + \boldsymbol{BA} + \boldsymbol{AB} + \boldsymbol{B}^2$$

Also, in case it is not immediately clear to you at this point, we can combine like terms as usual,

$$\boldsymbol{AB} + \boldsymbol{AB} = 2\boldsymbol{AB}$$

a. $\mathbf{A}(\mathbf{BC} - \mathbf{CD}) + \mathbf{A}(\mathbf{C} - \mathbf{B})\mathbf{D} - \mathbf{AB}(\mathbf{C} - \mathbf{D})$

$\cancel{ABC} - \cancel{ACD} + \cancel{ACD} - \cancel{ABD} - \cancel{ABC} + \cancel{ABD}$ $\boxed{= 0}$

b. $(\mathbf{A} - \mathbf{B})(\mathbf{C} - \mathbf{A}) + (\mathbf{C} - \mathbf{B})(\mathbf{A} - \mathbf{C}) + (\mathbf{C} - \mathbf{A})^2$ $(C-A)(C-A)$

$\cancel{AC} - \cancel{A^2} - \cancel{BC} + \cancel{BA} + \cancel{CA} - \cancel{C^2} - \cancel{BA} + \cancel{BC} + \cancel{C^2} - \cancel{CA} - \cancel{AC} + \cancel{A^2}$ $\boxed{= 0}$

c. $(\mathbf{A}^T\mathbf{C}^T)^T(\mathbf{CA}^T)^T(\mathbf{AC}^T)^T$

$(CA)(AC^T)(CA^T) = \boxed{CA^2C^TCA^T}$

d. $(\mathbf{I} - \mathbf{BA})(\mathbf{I} - \mathbf{BA}) + \mathbf{B}(2\mathbf{A} - \mathbf{ABA})$

$I - \cancel{2BA} + \cancel{BABA} + \cancel{2BA} - \cancel{BABA}$ $\boxed{= I}$

e. $\mathbf{A}^{-1}(\mathbf{B}^2\mathbf{A}^T)^T\mathbf{B}^{-T}$

$\underbrace{A^{-1}(A}_{I}\underbrace{B^TB^T)B^{-T}}_{I}$ $\boxed{= B^T}$