Worksheet - Lecture 2

Introduction to Linear Algebra Part Two

1. Use the following matrices or vectors to answer the following questions:

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 8 \\ 3 & 0 & -2 \\ 8 & -2 & -3 \end{pmatrix} \quad \mathbf{M} = \begin{pmatrix} 1 & 8 & -2 & 5 \\ 2 & 8 & 1 & 7 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} 6 \\ 3 \\ -1 \\ 2 \end{pmatrix} \quad \mathbf{u} = \begin{pmatrix} 6 & 4 & 8 & 1 \end{pmatrix}$$

a. Write the following matrices or vectors:

$\mathbf{M}^T =$ $\begin{pmatrix} 1 & 2 \\ 8 & 8 \\ -2 & 1 \\ 5 & 7 \end{pmatrix}$

$\mathbf{v}^T =$ $\begin{pmatrix} 6 & 3 & -1 & 2 \end{pmatrix}$

$\mathbf{A}^T =$ $\begin{pmatrix} 1 & 3 & 8 \\ 3 & 0 & -2 \\ 8 & -2 & -3 \end{pmatrix}$

$\mathbf{u}^T =$ $\begin{pmatrix} 6 \\ 4 \\ 8 \\ 1 \end{pmatrix}$

$(\mathbf{M}^T)^T =$ $\begin{pmatrix} 1 & 8 & -2 & 5 \\ 2 & 8 & 1 & 7 \end{pmatrix}$

b. Which of these matrices is symmetric? A, D

c. Can a rectangular matrix be symmetric? no. A^T cannot equal A unless these matrices have the same size!

d. What is $Tr(\mathbf{A})$?

$Tr(A) = 1 + 0 + (-3) = -2$

e. If possible, compute $Tr(\mathbf{M})$. If not possible, explain why.

not possible. Trace is only defined for square matrices

2. Write out the following matrices or vectors:

$\mathbf{I}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad \mathbf{e}_3 \in \mathbb{R}^5 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

$diag\{\sigma_1, \sigma_2, \sigma_3\} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$

3. If a matrix $\mathbf{M}$ is upper triangular than $\mathbf{M}^T$ is lower triangular

4. If $\mathbf{S}$ is a diagonal matrix, then $\mathbf{S}_{12} =$ 0