Worksheet - Lecture 5
Applications of Matrix Arithmetic

1. A florist offers three sizes of flower arrangements (small, medium, large) containing three types of flowers (roses, daisies, and chrysanthemums). The number of each type of flower in each size arrangement is given in the table below, along with the selling price of each arrangement and the cost of each individual flower.

	Roses	Daisies	Chrys.	Price
Small	1	3	3	$10
Medium	2	4	6	$15
Large	4	8	6	$20
Cost	$0.50	$0.25	$0.10	

Let

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 2 & 4 & 6 \\ 4 & 8 & 6 \end{pmatrix} \quad p = \begin{pmatrix} 10 \\ 15 \\ 20 \end{pmatrix} \quad c = \begin{pmatrix} 0.50 \\ 0.25 \\ 0.10 \end{pmatrix}.$$

   a. Determine the matrix-vector product that produces a vector, y which gives the total cost of creating each size arrangement (small, medium, and large).

   $$y = Ac$$

   b. Suppose that an order came in for 2 small arrangements and 2 large arrangements. Let

   $$v = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

   Using matrix arithmetic (and writing out the formula) determine both the price of this order and the total profit to the florist.

   $$price = v^T p \qquad (or \ equivalently \ p^T v)$$

   $$cost = v^T y = v^T A c$$

   $$\boxed{profit = v^T p - v^T A c}$$

2. Write the following system of equations as a matrix-vector product $\mathbf{Ax = b}$:

$$\begin{cases} \quad\; 2x_2 + 3x_3 = 8 \\ 2x_1 + 3x_2 + 1x_3 = 5 \\ \;\; x_1 - \;\; x_2 - 2x_3 = -5 \end{cases}$$

$$\underbrace{\begin{pmatrix} 0 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & -2 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 8 \\ 5 \\ -5 \end{pmatrix}}_{b}$$

3. A model is being developed to predict a student's SAT score based upon some numeric attributes. The data being used for this model is provided below:

Observation	PSAT score	Mother's SAT score	SAT Score
1	1600	1700	1750
2	1800	1250	1750
3	1750	1300	1600
4	1200	1800	1450
5	1350	1950	1500

If our regression model is

$$SAT\_score = \beta_0 + \beta_1 * PSAT\_score + \beta_2 * Mothers\_SAT\_Score + \epsilon$$

So how we'd set up the underlying matrix equation for regression analysis,

$$\mathbf{y = X\beta}$$

by defining the matrices/vectors $\mathbf{X, \beta}$, and $\mathbf{y}$.

$$X = \begin{pmatrix} 1 & 1600 & 1700 \\ 1 & 1800 & 1250 \\ 1 & 1750 & 1300 \\ 1 & 1200 & 1800 \\ 1 & 1350 & 1950 \end{pmatrix}$$

$$\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$

$$y = \begin{pmatrix} 1750 \\ 1750 \\ 1600 \\ 1450 \\ 1500 \end{pmatrix}$$