

Worksheet - Lecture 6

Gaussian Elimination

1. Using Gaussian Elimination on the augmented matrices, reduce each system of equations to a triangular form and solve using back-substitution.

a.

$$\begin{cases} x_1 + 2x_2 = 3 \\ -x_1 + x_2 = 0 \end{cases}$$

$$\left(\begin{array}{cc|c} 1 & 2 & 3 \\ -1 & 1 & 0 \end{array} \right) \xrightarrow{R_2' = R_2 + R_1} \left(\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 3 & 3 \end{array} \right) \rightarrow \begin{aligned} x_1 + 2x_2 &= 3 \\ 3x_2 &= 3 \\ \Rightarrow x_2 &= 1 \\ x_1 + 2(1) &= 3 \\ \Rightarrow x_1 &= 1 \end{aligned}$$

b.

$$\begin{cases} x_1 + x_2 + 2x_3 = 7 \\ x_1 + x_3 = 4 \\ -2x_1 - 2x_2 = -6 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 1 & 0 & 1 & 4 \\ -2 & -2 & 0 & -6 \end{array} \right) \xrightarrow{\substack{R_2' = R_2 - R_1 \\ R_3' = R_3 + 2R_1}} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array} \right) \rightarrow \begin{aligned} x_1 + x_2 + 2x_3 &= 7 \\ -x_2 - x_3 &= -3 \\ 4x_3 &= 8 \\ \Rightarrow x_3 &= 2 \\ -x_2 - (2) &= -3 \\ \Rightarrow x_2 &= 1 \\ x_1 + (1) + 2(2) &= 7 \Rightarrow x_1 = 2 \end{aligned}$$

c.

$$\begin{cases} 2x_1 - x_2 + x_3 = 1 \\ -x_1 + 2x_2 + 3x_3 = 6 \\ x_2 + 4x_3 = 6 \end{cases}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ -1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 6 \end{array} \right) \xrightarrow{R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|c} -1 & 2 & 3 & 6 \\ 2 & -1 & 1 & 1 \\ 0 & 1 & 4 & 6 \end{array} \right)$$

$$\xrightarrow{R_2' = R_2 + 2R_1} \left(\begin{array}{ccc|c} -1 & 2 & 3 & 6 \\ 0 & 3 & 7 & 13 \\ 0 & 1 & 4 & 6 \end{array} \right) \xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{ccc|c} -1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 6 \\ 0 & 3 & 7 & 13 \end{array} \right)$$

$$\xrightarrow{R_3' = R_3 - 3R_2} \left(\begin{array}{ccc|c} -1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & -5 & -5 \end{array} \right) \Rightarrow \boxed{x_3 = 1} \Rightarrow \boxed{x_2 = 2} \Rightarrow \boxed{x_1 = 1}$$