Worksheet - Lecture 7
Gauss-Jordan Elimination

1. Using Gauss-Jordan Elimination on the augmented matrices, reduce each system of equations from the previous worksheet (listed again below) to reduced row-echelon form and give the solution as a vector.

a. Starting from previous worksheet solution

$$\begin{cases} x_1 + 2x_2 = 3 \\ -x_1 + x_2 = 0 \end{cases}$$

$$\left(\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 3 & 3 \end{array}\right) \xrightarrow{R_2' = \frac{1}{3}R_2} \left(\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 1 \end{array}\right) \xrightarrow{R_1' = R_1 - 2R_2} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right)$$

$$\Rightarrow \boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

b.

$$\begin{cases} x_1 + x_2 + 2x_3 = 7 \\ x_1 + x_3 = 4 \\ -2x_1 - 2x_2 = -6 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right) \xrightarrow[R_2' = -1R_2]{R_3' = \frac{1}{4}R_3} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\xrightarrow[R_1' = R_1 - 2R_3]{R_2' = R_2 - R_3} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow{R_1' = R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\Rightarrow \boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}}$$

c.

$$\begin{cases} 2x_1 - x_2 + x_3 = 1 \\ -x_1 + 2x_2 + 3x_3 = 6 \\ x_2 + 4x_3 = 6 \end{cases}$$

$$\left(\begin{array}{ccc|c} -1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & -5 & -5 \end{array}\right) \xrightarrow[R_3' = -\frac{1}{5}R_3]{R_1' = -1R_1} \left(\begin{array}{ccc|c} 1 & -2 & -3 & -6 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow[R_1' = R_1 + 3R_3]{R_2' = R_2 - 4R_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{R_1' = R_1 + 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\Rightarrow \boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}}$$