Worksheet - Lecture 9
Solving For A Matrix Inverse

1. Find the inverse of the following matrices, using the method of Gauss-Jordan Elimination to solve the matrix equation $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$.

   a. $\mathbf{A} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

   $$\left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & -3/2 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right)$$

   $$A^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -3/2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

   b. $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 2 \end{pmatrix}$

   $$\left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1/2 \end{array}\right)$$

   $$\Rightarrow A^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1/2 \end{pmatrix}$$

2. What is the inverse of a full rank diagonal matrix,

   $$\mathbf{D} = diag\{d_{11}, d_{22}, \ldots, d_{nn}\}$$

   where each diagonal element, $d_{ii} \neq 0$?

   $$D^{-1} = diag\left\{\frac{1}{d_{11}}, \frac{1}{d_{22}}, \cdots, \frac{1}{d_{nn}}\right\}$$